Ron W. Nielsen

Evidence-based

UNIFIED GROWTH THEORY

Vol.2

Mechanism of the growth of population and of economic growth in the past 2,000,000 years explained

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Preface

The true laboratory is the mind, where behind illusions we uncover the laws of truth.
— Sir Jagadish Chandra Bose

Duration is not a test of true or false.
— Anne Morrow Lindbergh

If they don't depend on true evidence, scientists are no better than gossips.
— Penelope Fitzgerald

In science it often happens that scientists say, 'You know that's a really good argument; my position is mistaken,' and then they would actually change their minds and you never hear that old view from them again. They really do it. It doesn't happen as often as it should, because scientists are human and change is sometimes painful. But it happens every day.
— Carl Sagan

Unified Growth Theory[1] published by Oded Galor is called unified because it puts together earlier attempts to explain the historical economic growth and the historical growth of population. These attempts were made over many years and by now they form the established knowledge in economics and in demography.

Unfortunately, the past research was difficult because (1) access to data was strongly limited and (2) growth turns out to be represented by strongly deceptive distributions. They create an illusion of stagnation followed by a sudden explosion, while in fact they increase monotonically all the time and there is no sudden transition from a slow to fast growth. Data represented by these distributions have to be carefully and methodically analysed; otherwise conclusions are based on illusions.

Galor was in a far better position than many of the past researchers because he had access to superb and extensive sets of data made available by the world renown economist, Angus Maddison. These data describe economic growth and the growth of population, global, regional and even in individual countries. They

[1] Galor, O. (2011). *Unified Growth Theory*. Princeton, New Jersey: Princeton University Press.

are a rich source of information, which Galor failed to use. He made no attempt to analyse them.

There is no explanation for his neglect to analyse data mathematically because (1) he uses mathematics in his theory and thus he is familiar with mathematical procedures and (2) because trajectories describing growth of population and economic growth, while being deceptive, are trivially easy to analyse. No great skill is needed to analyse these distributions. Indeed, there is even no need to analyse them mathematically. Reliable conclusions can be reached just by using different plots of data. However, mathematical analysis, which is simple and easy, helps in a better understanding of the mechanism of growth.

Galor ignored also the earlier evidence published in 1960 that the growth of population during the AD section of time was hyperbolic. Using this information, the obvious next step would be to check whether the same type of growth is applicable to the economic growth.

Rather than using the previously published evidence, he systematically presented data in a suitably distorted way to support preconceived ideas. He could have made an important discovery but he did not. His theory presents nothing new. It is just a repetition of old interpretations of the growth of population and of economic growth, incorrect interpretations because they are contradicted by data. Unified Growth Theory is repeatedly contradicted even by the same data, which were used during its formulation.

The presented here *Evidence-based Unified Growth Theory* is firmly supported by a rigorous, mathematical analysis of data describing economic growth and the growth of population. It is also called *unified* because it presents a unified explanation of the growth of population and of economic growth in the past 2,000,000 years.

The terms *Malthusian stagnation, Malthusian regime* and *Malthusian trap* will be used in the presented here discussion but it should be remembered that they are incorrect, because Malthus never claimed that his positive checks were causing stagnation or creating a certain regime of growth or a trap. On the contrary, he observed that they stimulated growth and he even suggested that this curious phenomenon should be further investigated. Unfortunately, his observation was ignored, dubious concepts were later introduced and the name of Malthus was questionably attached to them, which Malthus would probably not approve. These phrases are used only because in this form, they are repeatedly used in the published literature.

This book is a compilation of my articles describing the investigation of the growth of population and of economic growth. I start by showing why the established knowledge is scientifically unacceptable. I follow this chapter by the introduction of a simple method of reciprocal values, which makes the analysis of hyperbolic distributions trivially simple. These two introductory chapters are followed by the explanation how the Unified Growth Theory is contradicted by data. These chapters are in turn followed by a detailed study of the growth of human population and of economic growth in the past 2,000,000 years; by the discussion of earlier attempts to explain the mechanism of hyperbolic growth; by the examination of the impacts of Malthusian positive checks; by the examination of impacts of demographic catastrophes; by the examination of the relation between the growth rate and growth trajectories, the essential step leading to the explanation of the mechanism of growth; by the formulation of the general law of growth; and by the explanation of the mechanism of the hyperbolic growth of human population and of the economic growth.

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Gold Coast, Australia
July, 2018

Contents

Introduction

This introduction is designed as a guide to the topics discussed in this book.

The spontaneous (unconstrained and undisturbed) growth of human population is not exponential, as was expected by Malthus, but hyperbolic. The same applies to the economic growth. This conclusion is in harmony with the earlier investigation carried out by von Foerster, Mora and Amiot[2] who studied the growth of the world population during the AD section of time. However, the study presented here extends the analysis to the BC time and to the economic growth. It also includes the analysis of regional growth of population and regional economic growth.

Results presented here are also in harmony with the earlier study of Deevey[3], who observed that growth of human population in the past 1,000,000 years was in three stages. However, he postulated that each stage was reaching an equilibrium. Results presented here confirmed the three stages of growth but demonstrated that each stage was hyperbolic. Rather than reaching an equilibrium, each stage had a potential to increase to infinity and was at a certain time terminated.

Two well-known theories, the Unified Growth Theory and the Demographic Transition Theory[4], are contradicted by the same data, which were used in their support.

[2] von Foerster, H., Mora, P., & Amiot, L. (1960). Doomsday: Friday, 13 November, A.D. 2026. *Science, 132*, 1291-1295.
[3] Deevey, E. S. Jr (1960). The human population. *Scientific American,203*(9), 195-204.
[4] For references see Chapter 5.

In the case of the Demographic Transition Theory, data, which appeared to be in support of this theory, were never analysed. Conclusions are based on impressions. However, in addition, contradicting data are systematically ignored.

In the case of the Unified Growth Theory, data were also never analysed but they were suitably distorted to support preconceived ideas. This deliberately distorted and misleading presentation of data is used in many other related publications.

There is no convincing explanation why the Author of the Unified Growth theory failed to analyse data mathematically and why he was systematically presenting them in a distorted way, because (1) he used mathematics in his theory and thus he is familiar with mathematical procedures, (2) hyperbolic growth was demonstrated as early as in 1960, (3) it is hard to imagine that he is not familiar with the fundamental properties of hyperbolic distributions, that they increase slowly over a long time and fast over a short time but that they increase monotonically, and (4) mathematical analysis of hyperbolic distributions is trivially simple.

Precisely the same data, which in their deliberately distorted way were used to support the Unified Growth Theory, are in fact in its direct contradiction. It is hard to understand why so much work was devoted to support the earlier erroneous interpretations of the mechanism of growth and why data were not properly analysed to check whether these interpretations, which were earlier based on limited data and on illusions, could be still supported.

Income per capita distributions show puzzling characteristics. They show that over a long time, income per capita was approximately constant but then, most recently, it was increasing extremely rapidly. The analysis of data presented here explains these puzzling characteristic features. They reflect nothing more than mathematical properties of dividing two hyperbolic distributions. They do not represent some peculiar mechanism applicable only to the economic growth but the feature, which applies to any two hyperbolic distributions, with only one condition that the singularity of the numerator is earlier than the singularity of the denominator.

Galor describes certain mysteries of growth in his Unified Growth Theory and indicates that they should be studied and explained. They have now been explained. They have nothing to do with the growth of population or with the economic growth. They were created by his distorted representations of data.

Galor describes a puzzling phenomenon of great divergence. This claimed phenomenon is also nothing more than a feature created by his distorted representations of data. There was no great divergence and there is nothing to explain except to explain how the great divergence was created by Galor.

Industrial Revolution had no impact on changing growth trajectories describing growth of population and economic growth, even in Western Europe and even in the United Kingdom. Forces associated with the Industrial Revolution are reflected in changing socio-economic conditions but they did not shape growth trajectories of the growth of population and of economic growth.

With the exception of just one event, demographic catastrophes had no impact on shaping the growth of population. The one and only exceptional event in the past 2,000,000 years, as presented by data, was an unusual convergence of five strong demographic catastrophes between AD 1195 and 1470. However, even this unusual event caused only a minor disturbance in the growth trajectory. When this exceptionally strong crisis was over, growth of population was even faster than before.

Survey of demographic catastrophes indicated that they were, in general, too weak to cause a major disruption in the growth of the world population even if they had strong local impacts. Analysis of Malthusian positive checks also added to the explanation why demographic catastrophes did not shape the growth of the world population.

It is interesting that Malthus noticed the dichotomous property of his positive checks, i.e. their destructive and regenerating effects. He even suggested that the regenerating effects should be further investigated. Unfortunately, the original observation of Malthus was ignored and the destructive aspect of his positive checks was blown out of proportion and used to explain the claimed prolonged stagnation, that never happened, while no effort was made to understand their regenerating property, which is in fact common in nature.

Mathematical analysis of the effects of Malthusian positive checks has now been carried out and it demonstrated that Malthus was right. His positive checks increase mortality rates but they also increase fertility rates, with the combined effect of increasing the growth rate. The regeneration process, or the growth stimulating property, is so efficient that the growth is even faster. This is a well-known phenomenon but it is an inconvenient property for those who created the concept of the prolonged epoch of stagnation used in the Demographic Growth Theory and in the Unified Growth Theory.

As a part of the presented here investigations, general law of growth was formulated and used to explain the mechanism of hyperbolic growth of population and of economic growth. It turns out that the mechanism is exceptionally simple, which is hardly surprising because hyperbolic growth is described by an exceptionally simple mathematical formula.

With the exception of two major transitions (46,000 - 27,000 BC and 425 BC – AD 510) and one minor disturbance (AD 1195 – 1470), growth of the world population in the past 2,000,000 years was consistently hyperbolic. It was steadily increasing without any signs of a random behaviour or of a sudden rapid increase towards the end of this long time. There was no stagnation and no sudden explosion. The same applies to the economic growth, which for the most part of the past 2,000,000 years was directly proportional to the size of human population. Explanation of the dynamics of growth is much simpler than presented in the Unified Growth Theory or in the Demographic Growth Theory or in many other published discussions, which ignore the earlier evidence of hyperbolic growth and which are not supported by a rigorous analysis of data but by impressions and conjectures.

1. Unified Growth Theory contradicted by the growth rate of income per capita

Introduction

In the subsection entitled "Mysteries of the growth process" (Galor, 2005a, p. 220) presented in his Unified Growth Theory (Galor, 2005a; 2011), Galor asks a series of questions about the mysteries of growth. We can take his questions one by one and show that all these mysteries were of his own creation.

His theory is *not* based on the scientific analysis of data but on impressions supported by the habitually distorted presentation of data (Ashraf, 2009; Galor, 2005a, 2005b; 2007; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). Such approach to research can easily create many mysteries that simply do not exist.

One of Galor's questions about the supposed mysteries of growth process is "What is the origin of the sudden spurt in growth rates of output per capita and population that occurred in the course of the take-off from stagnation to growth?" (Galor, 2005a, p. 220). In just one sentence, Galor presents *two* incorrect doctrines: the doctrine of the presence of the sudden spurts and the doctrine of transition from stagnation to growth, both created by the failure to follow scientific principles of investigation, which require that data should *not* be manipulated to support preconceived ideas but that they should be methodically analysed.

We shall show that this question makes as much sense as the question, "Why does the sun rotate around the earth?" and the answer to both of them is similar: The sun does not rotate around the earth and there was no sudden spurt in the growth rates of output per capita and population. There was also no takeoff from stagnation to growth because the growth was hyperbolic (Kapitza, 2006; Kremer, 1993; Nielsen, 2014; 2015; 2016a; 2016b; 2016c; 2016d; 2016e; Podlazov, 2002; Shklovskii, 1962; 2002; von Foerster, Mora & Amiot, 1960; von Hoerner, 1975). Hyperbolic growth is monotonic, and consequently it is also characterised by the monotonically-increasing growth rate. There is no sudden spurt in this type of distributions. These two doctrines expressed so confidently in just one sentence in Galor's publication are incorrect.

Output per capita (also described as income per capita and measured using the GDP/cap) is represented by the ratio of two, monotonically-increasing, hyperbolic distributions (Nielsen, 2015). The growth rate of this ratio is monotonic. It cannot contain "the sudden spurt" claimed erroneously by Galor.

Galor's questions about the mysteries of growth are strongly misleading because they describe features created by the repeatedly distorted presentations of data. The created features and the associated questions divert attention from the correct understanding of the mechanism of economic growth and of the growth of population. Galor's theory does not explain the mechanism of economic growth but describes phantom features he created.

We have already discussed (Nielsen, 2014; 2015; 2016a; 2016b; 2016c; 2016d) various aspects of Galor's theory (Galor, 2005a; 2011). We shall now focus our attention on the discussion of his unsubstantiated claims about the growth rates. We shall use precisely the same data (Maddison, 2001) as used by Galor (2005a; 2011), who unfortunately did not analyse them.

Unified Growth Theory is fundamentally incorrect but it is just an embodiment of the incorrect concepts used traditionally in the economic and demographic research, concepts developed by accretion over many years and now so strongly entrenched that it will be probably difficult to uproot them and replace them by correct interpretations. However, it is expected that it is in the interest of every economist and demographer to have scientific basis for their research.

These erroneous interpretations revolve around the concept of Malthusian stagnation and around the concept of transition from stagnation to growth. The study presented here and in earlier publications (Kapitza, 2006; Kremer, 1993; Nielsen, 2014; 2015; 2016a; 2016b; 2016c; 2016d, 2016e; Podlazov, 2002; Shklovskii, 1962; 2002; von Foerster, Mora & Amiot, 1960; von Hoerner, 1975) demonstrate that these traditional interpretations need to be replaced by interpretations based on the mathematical analysis of data and on the correct understanding of hyperbolic distributions.

The latest data of Maddison (2001; 2010) serve as a rich source of information. They are in perfect agreement with other similar data (e.g. Biraben, 1980; Clark, 1968; Cook, 1960; Durand, 1967; 1974; 1977; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994). When mathematically analysed, conclusions based on these data are also in harmony with earlier similar studies (Kapitza, 2006; Kremer, 1993; Podlazov, 2002; Shklovskii, 1962; 2002; von Foerster, Mora & Amiot, 1960; von Hoerner, 1975). Their combined message is that the demographic and economic research has to be based on accepting the unambiguous and consistent evidence in data that the historical growth of human population and of economic growth were hyperbolic and that such a growth cannot be divided into two or three different regimes of growth governed by distinctly different mechanisms of growth. Hyperbolic growth is slow over a long time and fast over a short time but it is still the same growth governed by the same mechanism of growth. Hyperbolic distributions have to be interpreted as a whole and not in parts. What appears as stagnation is hyperbolic growth and what appears as takeoff or explosion is the natural continuation of the same hyperbolic growth.

Fundamental mathematics

Growth rate $R(S)$ of a growing entity S can be defined as:

$$R(S) \equiv \frac{1}{S}\frac{dS}{dt} \quad , \tag{1}$$

Where S can represent the GDP, the size of the population or any other growing entity.

Let us assume that we have two growing entities S_1 and S_2, and that we want to calculate the growth rate of the ratio of these two entities, i.e. the growth rate $R(S_1/S_2)$. It is easy to see that

$$R(S_1/S_2) \equiv \frac{1}{S_1/S_2}\frac{d(S_1/S_2)}{dt} = R(S_1) - R(S_2) \, . \tag{2}$$

We have obtained an interesting equation: the growth rate of the ratio of two distributions is the difference between the growth rates of its two components. Thus, for instance, the growth rate of the GDP/cap is given by the difference between the growth rate of the GDP and the growth rate of population.

If two growing entities increase monotonically (as it is in the case of the historical economic growth and of the historical growth of population) their growth rates also increase monotonically and consequently the growth rate of their ratio, which is represented by the difference between the monotonically-increasing growth rates of each of the two components, is also increasing monotonically. It does not contain a sudden spurt.

Hyperbolic growth is described by the following simple differential equation:

$$\frac{1}{S}\frac{dS}{dt} = kS \,, \qquad (3)$$

Where S can represent the GDP or the size of the population, or indeed any other hyperbolically-increasing entity, while k is a positive constant.

If we compere this differential equation with the general definition of the growth rate given by the eqn (1) we can see that the characteristic feature of hyperbolic growth is that its growth rate is directly proportional to the size of the growing entity:

$$R(S) = kS \,. \qquad (4)$$

The growth rate of hyperbolic distributions increases hyperbolically. The time dependence of the growth rate of hyperbolic distributions creates precisely the same illusions as the time dependence of hyperbolic growth (Nielsen, 2014). The growth rate of hyperbolic distribution is slow over a long time and fast over a short time but it is a monotonically-increasing distribution, which cannot be divided into mathematically-justifiable slow and fast components because the transition from slow to fast growth occurs all the time along the entire time-range of such a distribution. The growth rate of hyperbolic growth does not contain any sudden spurt at any time.

The equation (3) can be solved easily by substitution $S = Z^{-1}$. Its solution is:

$$S = \frac{1}{(a-kt)} \,, \qquad (5)$$

Where a is a constant, which can be determined empirically by comparing the calculated curve with data.

So, now, if we use the eqn (2) and if we assume that S_1 and S_2 are hyperbolic, then

$$S_1 = \frac{1}{(a_1 - k_1 t)} \tag{6}$$

and

$$S_2 = \frac{1}{(a_2 - k_2 t)} . \tag{7}$$

Consequently, by using the eqn (4) we have

$$R(S_1) = \frac{k_1}{a_1 - k_1 t} \tag{8}$$

and

$$R(S_2) = \frac{k_2}{a_2 - k_2 t} . \tag{9}$$

If we now use these expressions in the eqn (2) we shall get

$$R(S_1/S_2) = \frac{k_1}{a_1 - k_1 t} - \frac{k_2}{a_2 - k_2 t} = \frac{\Delta}{(a_1 - k_1 t)(a_2 - k_2 t)} , \tag{10}$$

where

$$\Delta = k_1 a_2 - k_2 a_1 . \tag{11}$$

The eqn (10) can be also presented as

$$R(S_1 / S_2) = \frac{1}{A_0 + A_1 t + A_2 t^2} , \tag{12}$$

where

$$A_0 = \frac{a_1 a_2}{\Delta} , \tag{13}$$

$$A_1 = -\frac{k_1 a_2 + k_2 a_1}{\Delta} , \tag{14}$$

$$A_2 = \frac{k_1 k_2}{\Delta}. \qquad (15)$$

So, while the growth rate of hyperbolic distributions is described by the reciprocal of a linear function [see the eqns (8) and (9)] the growth rate of the ratio of hyperbolic distributions is described by the reciprocal of the second-order polynomial [see the eqn (12)]. We could call it the second-order hyperbolic distribution. It is a distribution, which resembles closely the first-order hyperbolic distribution (the reciprocal of the linear function) because it also increases slowly over a long time and escapes to infinity at a fixed time. However, it is a monotonically-increasing distribution, which does not contain a sudden spurt.

It obviously makes no sense to claim a sudden spurt in the monotonically changing second-order polynomial distribution and it obviously makes no sense to claim a sudden spurt in the reciprocal of the second-order polynomial. The sudden spurt can be created by distorting data, as Galor did, but then it is no longer science. Whether created on purpose to support preconceived ideas or carelessly, a similar distorted presentation of evidence would be unacceptable even outside science. However, distortion of evidence is not uncommon in defending doctrines accepted by faith. The distorted presentation of empirical evidence makes the Unified Growth Theory (Galor, 2005a; 2011) scientifically unacceptable.

Another way to understand that the growth rate of the ratio of two hyperbolic distributions (e.g. the growth rate of the GDP/cap distribution) cannot contain a sudden spurt is by looking at the denominator of the eqn (10), which is given by a product of two linearly decreasing functions. Multiplication of two linear distributions produces a monotonic distribution, which does not contain a sudden spurt.

Had Galor analysed the data (Maddison, 2001) he was using, he would have perhaps found that the GDP and the size of the population were increasing hyperbolically. Maybe, then, it would have been clear to him that monotonic distributions cannot be characterised by the non-monotonic sudden spurts. Such an analysis should have been prompted by the discovery made over 50 years ago that the growth of human population during the AD era was hyperbolic (von Foerster, Mora & Amiot, 1960).This vital discovery, published in a well-known journal, is not even mentioned in Galor's theory, maybe because it was an inconvenient discovery. Such an analysis should have been also prompted by other related studies (Kapitza, 2006; Kremer, 1993;

Podlazov, 2002; Shklovskii, 1962; 2002; von Hoerner, 1975). Finally, it should have been prompted by the data describing the historical growth of human population (e.g. Biraben, 1980; Clark, 1968; Cook, 1960; Durand, 1967; 1974; 1977; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994). All these works have been ignored and a scientifically unreliable theory has been created with many mysteries of the growth process, mysteries that do not exist.

As in the case of hyperbolic distributions which can be studied easily by investigating the reciprocal values of the size of the growing entity, $1/S$ (Nielsen, 2014), an easy way to study the growth rate of the ratio of hyperbolic distributions is by using the reciprocal values of the growth rate, $1/R(S_1/S_2)$. As $1/S$ converts the confusing hyperbolic distribution to a linear function, which is then easy to understand, so also the reciprocal values, $1/R(S_1/S_2)$, convert the second-order hyperbolic distribution into an easy-to-interpret second-order polynomial. In both cases, the confusing features, which create the illusion of a slow growth over a long time followed by a sudden spurt disappear and are replaced by much simpler distributions.

Confusing features of hyperbolic distributions can be also clarified by using semi logarithmic displays. Such displays are routinely used for distributions, which vary over a large range of values. We shall use them in our present discussion.

World economic growth

Growth of the GDP, population and income per capita (GDP/cap)

According to Galor, historical economic growth is characterised by takeoffs from stagnation to growth, which occurred around AD 1750 for developed regions and around AD 1900 for less-developed regions (Galor, 2008a; 2012a). He describes them as "stunning" or "remarkable" escapes from the Malthusian trap (Galor, 2005a, pp. 177, 220). Such remarkable escapes should be readily identifiable in data describing economic growth and the growth of human population. In particular, for data describing the world economic growth and the growth of population, we should see clearly two takeoffs, around AD 1750 and around AD 1900.

Results of our analysis of precisely the same data (Maddison, 2001) as used, but never analysed, by Galor during the formulation of the Unified Growth Theory (Galor, 2005a; 2011), are presented in Figures 1-3.

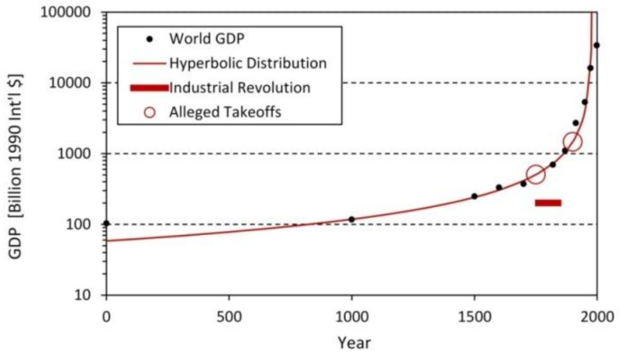

Figure 1. *Data for the Gross Domestic Product (Maddison, 2001), precisely the same data as used but never analysed by Galor during the formulation of the Unified Growth Theory (2005a; 2011), are compared with the first-order hyperbolic distribution [eqn (5)]. The GDP is expressed in billions of 1990 International Geary-Khamis dollars. Parameters describing the fitted hyperbolic distribution are:*
$a = 1.716\times10^{-2}$ *and* $k = 8.671\times10^{-6}$

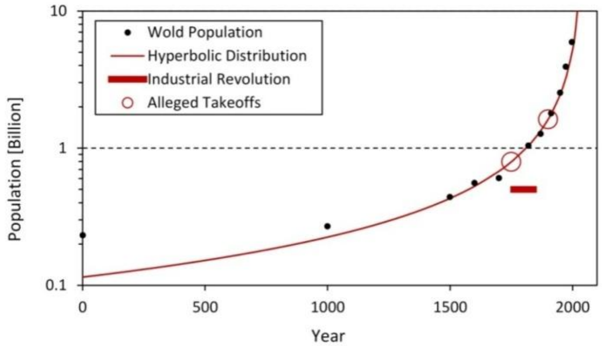

Figure 2. *Data describing the growth of the world population during the AD era (Maddison, 2001) are compared with hyperbolic distribution. The large discrepancy at AD 1 is because of the maximum in the growth of the world population around that year associated with the transition from a fast-increasing hyperbolic distribution during the BC era to a slower hyperbolic distribution during the AD era (Nielsen, 2016b). Parameters describing the fitted hyperbolic distribution are* $a = 8.724\times10^{0}$ *and* $k = 4.267\times10^{-3}$.

The supposed takeoff from the assumed stagnation to growth for developed countries coincides with the onset of the Industrial Revolution, AD 1760-1840 (Floud & McCloskey, 1994), which according to Galor was "the prime engine of economic growth" (Galor, 2005a, p. 212). It is, therefore, yet another reason why the takeoff for developed countries and the associated "sudden spurt" in the growth rates should be easy to identify because the supposed prime engine should have been working most effectively in these countries.

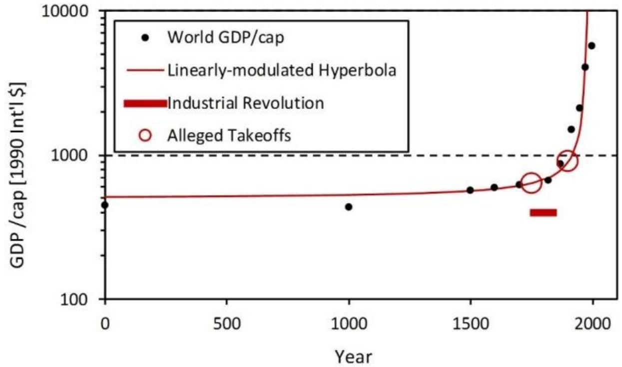

Figure 3. *World income per capita (GDP/cap). Data (Maddison, 2001) are compared with the linearly-modulated hyperbolic distribution (Nielsen, 2015) representing the ratio of hyperbolic distributions of the GDP and of the size of population. Income per capita was increasing monotonically. Such monotonic increase cannot produce a non-monotonic growth rate claimed by Galor (2005a; 2011). It cannot produce "the sudden spurt in the growth rates of output per capita" (Galor, 2005a, p. 220). His "stunning" or "remarkable" takeoffs from stagnation to growth (Galor, 2005a, pp. 177, 220) did not happen. Industrial Revolution, the "prime engine of economic growth" (Galor, 2005a, p. 212) had no impact on changing the economic-growth trajectory. All these stories are contradicted by data (Maddison, 2001), precisely the same data as used but never analysed during the formulation of the Unified Growth Theory.*

Economic growth, as described by the Gross Domestic Product (GDP) shown in Figure 1, was hyperbolic. The supposed "prime engine of economic growth" (Galor, 2005a, p. 212) did nothing to change the growth trajectory. This is an interesting issue, which requires further investigation because technological discoveries were used to support economic growth but surprisingly perhaps they had absolutely no impact on changing the growth trajectory. It is as if economic growth was controlled by some other unknown and much stronger force, which was active before the Industrial Revolution and remained undisturbed during and after the Industrial Revolution. The supposed takeoffs from stagnation to growth did not happen because there was no stagnation. Economic growth was hyperbolic before and after the supposed takeoffs. The takeoffs claimed by Galor simply did not exist.

The growth of population during the AD era, shown in Figure 2, was also hyperbolic, at least from around AD 1000, in perfect agreement with the discovery made over 50 years ago by von Foerster, Mora and Amiot (1960). The discrepancy at AD 1 is explained by the analysis of the growth of human population in the past 12,000 years (Nielsen, 2016b), which revealed a maximum around that year associated with the transition from a fast-increasing hyperbolic growth during the BC era to a slower hyperbolic growth during the AD era. This extended analysis demonstrated that there was an uninterrupted hyperbolic growth between 10,000 BC and around 500 BC, followed by a transition to a new hyperbolic growth commencing around AD 500. It also revealed a small disturbance of the hyperbolic growth between AD 1200 and 1400. The data show that during the past 12,000 years there was no stagnation and no sudden takeoff at any time, both in the growth of the population and in the growth rate.

It is remarkable that so many independent studies are in such perfect agreement: Maddison's data (Maddison, 2001, 2010) and their analysis (Nielsen, 2014; 2015; 2016a; 2016c; 2016d); the estimates of the size of human population not only during the AD era but also during the BC era (e.g. Biraben, 1980; Clark, 1968; Cook, 1960; Durand, 1967; 1974; 1977; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994) and their analysis (e.g. Kremer, 1993; Nielsen, 2016b; Kapitza, 2006); the discovery made by von Foerster, Mora and Amiot (1960) and similar identifications of hyperbolic growth by Podlazov (2002), Shklovskii (1962; 2002) and von Hoerner (1975).

In contrast, Unified Growth Theory and the generally accepted but questionable postulates used in the economic and demographic research describe events and processes occurring in the fictitious world characterised by Malthusian stagnation, takeoffs, sudden spurts and by the "remarkable" or "stunning" escapes from the Malthusian traps (Galor, 2005a, pp. 177, 220), the world which is entirely different than the world revealed by data and by their mathematical analysis.

There appears to be no formal definition of Malthusian stagnation. Maybe some kind of stagnation can be used to describe social conditions but this concept is totally irrelevant in the study of the mechanism of economic growth and of the growth of human population because they were hyperbolic. There was no stagnation and no transition from the imagined stagnation to growth. Using such descriptions to explain the mechanism of the historical economic growth or the historical growth of population is unscientific because these postulates are consistently contradicted by data.

Results of analysis of income per capita (GDP/cap) presented in Figure 3 also demonstrate a monotonically-increasing distribution at least from AD 1500, i.e. during the time when Galor's "remarkable" and "stunning" effects should be clearly visible. What is remarkable about this distribution is that nothing remarkable or stunning ever happened. The growth of the GDP/cap was remarkably stable. Industrial Revolution did not accelerate the growth of income per capita and there were no sudden takeoffs at any time.

Such monotonically-increasing distributions, as presented in Figures 1-3, cannot generate "the sudden spurt" (Galor, 2005a, p. 220) in the corresponding growth rates and we shall now see that they indeed did not.

Growth rates of the GDP, population and income per capita (GDP/cap)

Results of analysis of growth rates are shown in Figures 4-6. Empirical growth rates were calculated using Maddison's data (Maddison, 2001) and interpolated gradients. The predicted growth rates were calculated using the fitted distributions shown in Figures 1-3.

As expected, the growth rate of the world GDP and population were increasing monotonically. Industrial Revolution did not accelerate their growth. The "remarkable" or "stunning" escapes from the Malthusian trap (Galor, 2005a, pp. 177, 220), which were supposed to have been reflected in takeoffs from stagnation to growth, never happened because there was no stagnation and the trap did not exist in the economic growth and in the growth of population. Growth rates were increasing along remarkably robust trajectories.

Analysis of growth rates shows a remarkable contradiction of Galor's claims by *precisely the same data*, which he used, but never analysed, during the formulation of his Unified Growth Theory. His wished-for and claimed features never happened. Growth rates were increasing *monotonically*. There was absolutely no sudden spurt at any time.

In order to support his preconceived ideas, Galor ignored not only the analysis carried out over 50 years ago (von Foerster, Mora & Amiot, 1960) but also research contributions of his many other predecessors (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1967; 1974; 1977; Gallant, 1990; Haub, 1995; Kapitza, 2006; Kremer, 1993; Livi-Bacci, 1997; McEvedy & Jones, 1978; Podlazov, 2002; Shklovskii, 1962; 2002; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; von Hoerner, 1975). Galor's claims are in conflict with science. They are not just unsupported by science – they are repeatedly *contradicted* by the scientific analysis of data and most notably by the analysis of precisely the same data, which he used during the formulation of his theory.

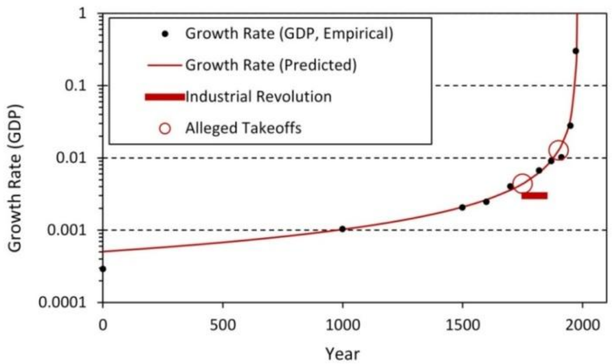

Figure 4. *Growth rate of the world GDP was increasing monotonically. There was no sudden spurt. The claimed takeoffs did not happen. Industrial Revolution did not accelerate economic growth.*

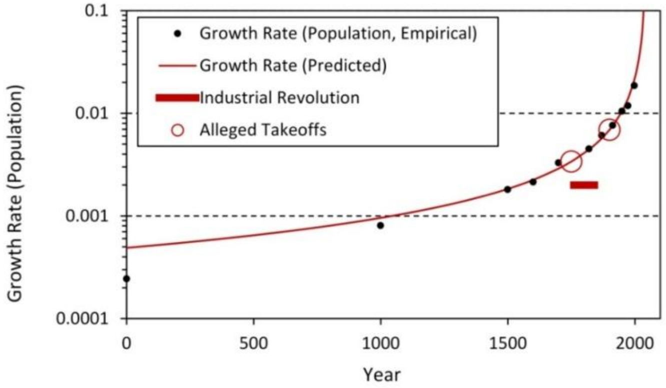

Figure 5. *Growth rate of the world population. Empirical growth rate calculated using Maddison's data (Maddison, 2001) and interpolated gradients is compared with the predicted growth rate calculated using parameters of the fitted hyperbolic distribution displayed in Figure 2. Galor's claims (Galor, 2005a; 2011) are remarkably contradicted by the analysis of Maddison's data (Maddison, 2001),* precisely the same data, *which he used but never properly analysed. Galor's mystery of "the sudden spurt" in the growth rate of population (Galor, 2005a, p. 220) is solved – there was no sudden spurt.*

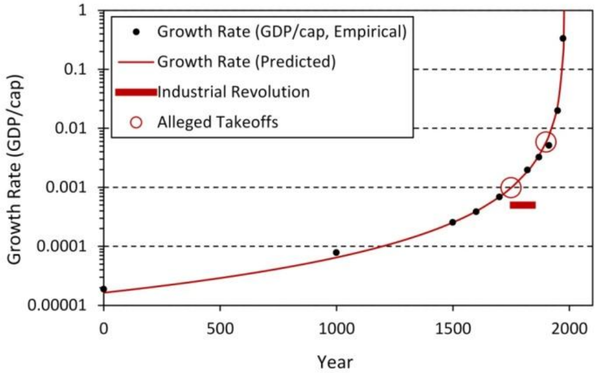

Figure 6. *Growth rate of the world income per capita (GDP/cap).Empirical growth rate calculated using Maddison's data (Maddison, 2001) and interpolated gradients is compared with the predicted growth rate calculated using parameters of the fitted hyperbolic distributions displayed in Figures 1 and 2. Galor's claims (Galor, 2005a; 2011) are remarkably contradicted by the analysis of Maddison's data (Maddison, 2001),* precisely the same data*, which he used but never properly analysed. Galor's mystery of "the sudden spurt" in the growth rate of income per capita (Galor, 2005a, p. 220) is solved – there was no sudden spurt.*

Mathematical analysis of Maddison's data (Maddison, 2001), precisely the same data as used by Galor, solves also his mystery "of the sudden spurt in growth rates of output per capita" (Galor, 2005a, p. 220) – there was no spurt. Results of analysis are presented in Figure 6. Growth rate of income per capita (GDP/cap) was increasing monotonically. Industrial Revolution did not accelerate the growth of income per capita. The postulated takeoffs from stagnation to growth (yet another mystery of growth claimed by Galor) did not happen because there was no stagnation and because the growth rate was increasing steadily without any major interruption. The only real mystery is why the growth rate of income per capita was so remarkably stable over such a long time.

Hyperbolic distributions, which increase monotonically, are characterised by monotonically-increasing growth rates, as shown in Figures 5-6. Claiming the existence of sudden spurts in such distributions is scientifically unjustifiable. Going a step further and claiming that such an imaginary spurt is a mystery, which needs to be explained encourages other researchers to carry out pointless and unproductive research.

It is useful to compare the mathematical analysis of Maddison's data presented in Figure 6 with the distorted presentation used by

Galor reproduced in Figure 7. Both figures are based on *precisely the same set of data* (Maddison, 2001). The contrast is striking. Now we can better appreciate the lack of scientific support for his theory. It is surprising that his theory was ever published. It is also surprising that similar distorted diagrams and the associated unscientific claims and explanation were published in peer-reviewed literature (Galor, 2005b; 2007; 2010; 2011; 2012b; Galor & Moav, 2002; Snowdon & Galor, 2008)

While data and their analysis, displayed in Figure 6, present monotonically-increasing growth rate of income per capita, Galor's distorted presentation of precisely the same data show a clear "sudden spurt." Maybe Galor was so strongly guided by the traditional interpretations of economic growth that he could not accept the clear contradicting evidence or maybe he simply did not know how to analyse data. In any case, intentional or unintentional, such ubiquitous distorted diagrams used repeatedly in his theory can be hardly expected to lead to reliable conclusions. On the contrary, they can be expected to lead only to incorrect conclusions.

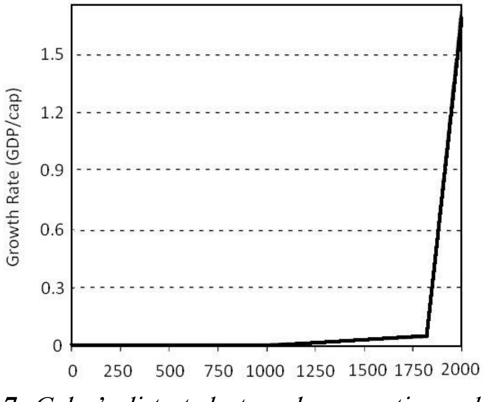

Figure 7. *Galor's distorted, strongly suggestive and misleading presentation of Maddison's data (Maddison, 2001) describing the growth rate of output (income) per capita (Galor, 2005a, p. 179). Precisely the same data, when correctly displayed and analysed (see Figure 6), show that" the sudden spurt in the growth rate of output per capita" claimed by Galor (2005a, p. 220) did not exist.*

Galor gives also many isolated examples of small growth rates in the past and significantly larger values at a later stage of growth but all these examples are not only meaningless but also strongly misleading. They reflect nothing more than just the natural properties of hyperbolic distributions. Using them to prove stagnation and transitions from stagnation to growth is scientifically irresponsible.

Of course, growth rates of income per capita (GDP/cap) were small over a long time and significantly larger at a certain later stage of growth because they were following monotonically-increasing second-order hyperbolic distributions [see eqn (12)]. Hyperbolic distributions (second-order or first-order) are slow over a long time and fast over a short time but they are still the same, monotonically-increasing distributions. They are not characterised by sudden spurts. There is no profound mystery about them that needs to be explained by some elaborate research or mathematical formulations. It is just a simple and straightforward hyperbolic growth. The mystery is solved. Picking up some isolated numbers from such hyperbolic distributions and drawing some profound conclusions based on such examples is unscientific. The only mystery that needs to be explained is why the economic growth and the growth of population were hyperbolic and why they were so remarkably stable (undisturbed) over such a long time.

Summary and conclusions

Galor discovered many "mysteries of the growth process" (Galor, 2005a, p. 220). One of his mysteries is "the sudden spurt in growth rates of output per capita and population that occurred in the course of the take-off from stagnation to growth" (Galor, 2005a, p. 220).

His discoveries are based on the crude and distorted presentations of data (Ashraf, 2009; Galor; 2005a; 2005b; 2007; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). His mysteries are of his own creation. They do not need to be explained because they do not exist. They describe the world of fiction.

Historical economic growth and the growth of human population were hyperbolic (Kapitza, 2006; Kremer, 1993; Nielsen, 2014; 2016a; 2016b; 2016c; 2016d; Podlazov, 2002; Shklovskii, 1962; 2002; von Foerster, Mora & Amiot, 1960; von Hoerner, 1975). Hyperbolic distributions are monotonic and they are characterised by the monotonically-increasing growth rates. Sudden spurts do not exist in such distributions.

It is essential to understand that it is incorrect to take a slowly-increasing distribution and automatically claim the evidence of Malthusian stagnation. The state of stagnation might occur when there is a strong interference between a primary force propelling growth and some random opposing forces. Effects of stagnation should be reflected in the growth trajectory, which should be clearly unstable. If the growth follows a steadily-increasing trajectory without any clear signs of random behaviour, then there is no need to complicate the description of the mechanism of growth by introducing random forces whose presence is undetectable. The fundamental principle of scientific investigation is to look for the simplest descriptions and explanations. Introducing unnecessary complications is simply unscientific.

The established knowledge in demography and in economic research is strongly based on a series of postulates revolving around the concept of Malthusian stagnation and around the supposed transition from the imagined stagnation to growth. Complicated mechanisms and interactions are then used (and gradually made even more complicated) to explain the mechanism of growth. Galor went one step further and reinforced these incorrect interpretations by his persistently distorted presentations of data (such as shown in Figure 7) and by his repeated quotations of certain well-selected figures to support his preconceived ideas, figures which were supposed to illustrate the concepts of stagnation and takeoffs from stagnation to growth but when closely analysed illustrate nothing else than the simple hyperbolic growth. All such complicated explanations are contradicted by data. Close examination of data shows that there was no stagnation and no transition from stagnation to growth. Data show consistently that the mechanism of the economic growth and of the growth of human population must have been simple because hyperbolic growth is exceptionally simple [see eqn (5)].

Some types of growth might be slow and stagnant but hyperbolic growth is not stagnant even when it is slow. It is prompted by the same mechanism during the time when it is slow and when it is fast. If the mechanism of Malthusian stagnation is used to explain the slow hyperbolic growth, precisely the same mechanism should be used to explain the fast growth, which is commonly described as explosion. It is incorrect to divide hyperbolic distributions into two or three sections and assign different mechanisms of growth to each of such arbitrarily selected sections. Hyperbolic distributions have to be explained as a whole and the same mechanism should be applied to the apparent slow and to the apparent fast sections.

It is incorrect to take a hyperbolic distribution and look for a sudden takeoff from the imagined stagnation to growth, as Galor did repeatedly. It is impossible to divide hyperbolic distribution into such distinctly different sections and the best way to see it, is by using the reciprocal values (Nielsen, 2014) because hyperbolic distribution is then represented by a decreasing straight line and it is obvious that it is impossible to claim a change of direction on the straight, which shows no change of direction.

Hyperbolic growth is not the only type of growth that can be slow over a certain time but not stagnant. Exponential growth is initially slow but it gradually becomes faster. At a certain stage, as if suddenly, it becomes overwhelmingly fast, the effect described as “the second half of the chessboard” (Kurzweil, 1999). Logistic growth is also initially slow but it is not stagnant.

The difference between hyperbolic and exponential distributions is that for hyperbolic distributions the apparent (but non-existent) transition from a slow to fast growth is more clearly articulated. That is why hyperbolic distributions are so often misinterpreted, particularly if they are distorted as it is done repeatedly and persistently in Galor’s publications. However, this apparent transition from slow to fast growth does not happen at any given time or even over a certain specific range of time. It happens all the time. The acceleration is gradual along the entire range of hyperbolic distribution.

Growth of income per capita (GDP/cap) is represented by the ratio of two hyperbolic distributions (Nielsen, 2015). The ratio of monotonically increasing- distributions is characterised by the monotonically-increasing growth rate. We have shown that while the growth rate of the GDP and population increases hyperbolically, the growth rate of income per capita (GDP/cap) increases by following a second-order hyperbolic distribution (the reciprocal of the second order polynomial). There is no sudden spurt in any of these distributions and we have demonstrated this point by using the world economic growth and the growth of human population.

When doctrines accepted by faith are defended, scientific rules of engagement are readily violated. Contradicting data are then either ignored or manipulated to support preconceived ideas. Economic and demographic research has no place for this type of activities. However, intentionally or unintentionally, such unscientific approach to research appears to have been adopted during the formulation of the Unified Growth Theory (Galor, 2005a; 2011). Numerous preceding research works (e.g. Biraben, 1980; Clark,1968; Cook,1960; Durand,1967; 1974; 1977; Gallant, 1990; Haub, 1995; Kapitza, 2006; Kremer, 1993; Livi-Bacci, 1997; McEvedy & Jones, 1978; Podlazov, 2002; Shklovskii, 1962; 2002; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; von Foerster, Mora & Amiot, 1960; von Hoerner, 1975) were ignored and the excellent data of Maddison (2001) were manipulated and distorted to support a series of preconceived ideas.

Hyperbolic distributions may be confusing but there is no excuse for distorting them to make them even more confusing. There is also no excuse for failing to analyse hyperbolic distributions because their analysis is trivially simple (Nielsen, 2014). The analysis of the growth rates is in the same category. Graphically, all these distributions become abundantly clear by using either the semi logarithmic scales of reference of by displaying the reciprocal values of growing entities or of their corresponding growth rates.

Galor's Unified Growth Theory is scientifically unacceptable and so are also many traditional interpretations of economic growth and of the growth of human population, interpretations based on the incorrect understanding of the mathematical properties of hyperbolic distributions. The recent and readily-accessible Maddison's data (Maddison, 2001; 2010) make it now easy to study the mechanism of the historical economic growth and of the growth of human population. They demonstrate that certain fundamental postulates revolving around the concept of Malthusian stagnation and around the assumed transition from the non-existent stagnation to growth, still used routinely in the established knowledge in demography and in economic research, are no longer acceptable.

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2. Unified Growth Theory contradicted by the absence of the great divergence

Introduction

Those who are less familiar with the scientific process of investigation might not be aware that there is also unscientific approach, which unfortunately appears to be used sometimes even in academic circles. It is important to have a clear understanding of these two different ways of investigation in order to be able to distinguish between acceptable and unacceptable claims and conclusions.

In science, theories are tested by data. In unscientific discussions, data are tested by theories. In unscientific presentations, selective use of data is common. Data are manipulated, distorted or rejected if they do not agree with preconceived ideas.

In the scientific research, contradicting evidence is not only accepted but looked for because it usually leads to new discoveries. In unscientific discussions, contradicting evidence is studiously rejected because it threatens the established knowledge.

In science, data are rigorously analysed. In non-scientific discussions, rigorous analysis is avoided and interpretations of data are based on impressions, but impressions can be misleading and even great thinkers can make a mistake. "It is clear that the earth does not move, and that it does not lie elsewhere than at the centre" (Aristotle). Appearances and logical explanations are not necessarily reliable. "Plato is my friend, Aristotle is my friend, but my greatest friend is truth" (Sir Isaak Newton).

It is also important to understand the limitations of mathematics. Elaborate stories and explanations can be translated into mathematical language but such translations are meaningless unless they can be tested by data.

We should never be mesmerised by complicated mathematical formulae and presentations. The essential question is whether the presented mathematics can be tested by relevant data. If stories translated into mathematics cannot be tested by data, if they have to be accepted by faith, then obviously they have no scientific value and they can be ignored or even rejected. Mathematical formulations should be making testable predictions. A story dressed up in a mathematical gown will be just a story unless it makes a testable prediction.

A good example of the unscientific approach to research is the Unified Growth Theory (Galor, 2005a; 2011). Data are manipulated and distorted. Selected data, which appear to support preconceived concepts, are repeatedly quoted. Excellent data of Maddison (2001) are used during the formulation of this theory but they are never analysed. They are presented in distorted ways to support preconceived ideas. Galor translates his assumed and scientifically-unsupported interpretations of economic growth into many complicated but rather primitive mathematical formulae. However, he does not make even a single mathematical prediction, which can be tested directly by data. His mathematical expressions do not describe growth trajectories that could be compared with data, even with data he uses during the formulation of his theory. Ironically, *precisely the same data*, when analysed, are in direct contradiction of his theory.

His concepts can be only tested indirectly by showing that within the range of the mathematically-analysable data there was no stagnation, no sudden takeoffs, no “remarkable” or “stunning” escapes from the Malthusian trap (Galor, 2005a, pp. 177, 220) and no transition from stagnation to the so-called sustained growth regime (Nielsen, 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2015h). Economic growth in the past was sustainable and secure, as indicated by the steadily-increasing hyperbolic trajectories, but now it is unsustainable and insecure (Nielsen, 2015b). The numerous mathematical formulae used in the Unified Growth Theory do not describe or explain the historical economic growth because they incorporate concepts, which are either contradicted repeatedly by data or have to be accepted by faith.

We have already demonstrated (Nielsen, 2014; 2015a; 2016a, 2016d; 2016e; 2016f; 2016g; 2016h) that Galor’s Unified Growth

Theory is fundamentally incorrect because it is based on fundamentally-incorrect ideas. We have shown that within the range of the mathematically-analysable data, historical economic growth and the growth of population were hyperbolic. For the economic growth, the range of evidence is limited but for the growth of human population it can be extended to 10,000 BC (Nielsen, 2016b). We have demonstrated that within the range of analysable data, there was no Malthusian stagnation and no Malthusian trap in economic growth and in the growth of population. The growth was slow over a long time but it was steadily increasing and there was no transition at any time in the past that could be described as a sudden takeoff, spurt, sprint or explosion. We have demonstrated that Galor's claim of sudden takeoffs is repeatedly contradicted by data. There were no takeoffs and consequently there was also no differential timing of takeoffs. During the time of the claimed takeoffs, economic growth and the growth of population were either continuing to increase along undisturbed and remarkably stable hyperbolic trajectories or they were diverted to *slower* trajectories. This conclusion applies not only to the growth of the Gross Domestic Product (GDP) and population but also to the growth of income per capita (GDP/cap). We do not have to try to explain the mechanism of the epoch of Malthusian stagnation and of the escape from the Malthusian trap because there was no stagnation and no trap in the economic growth and in the growth of population. What we have to explain is why the growth in the past was hyperbolic, why it was so remarkably stable and why it started to be diverted recently to new, non-hyperbolic trajectories.

The concept of the great divergence

The concept of the great divergence belongs to a set of other phantom "mysteries about the growth process" (Galor, 2005a, p. 220) invented by Galor and reinforced by the habitually distorted presentations of data (Ashraf, 2009; Galor, 2005a; 2005b; 2007; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). One example of such distorted presentation of data used routinely by Galor is shown in Figure 1. In contrast, the accurate presentation of *precisely the same data*, together with their mathematical analysis, is shown in Figure 2.

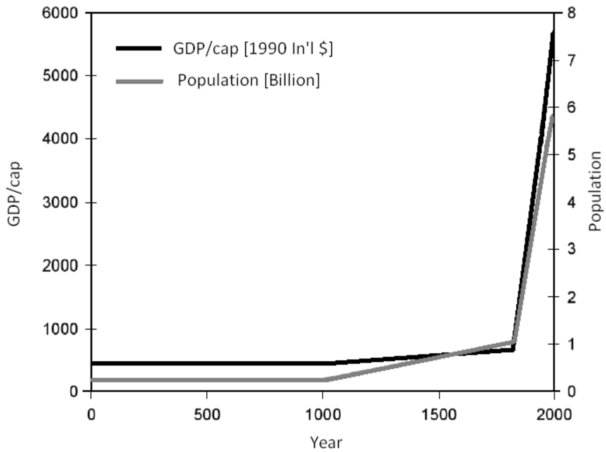

Figure 1. *Example of the ubiquitous, grossly-distorted and self-misleading diagrams used to create the Unified Growth Theory (Galor, 2005a; 2011). Maddison's data (Maddison, 2001) were used during the formulation of this theory but they were never analysed. Such state-of-the-art was used to construct a system of scientifically-unsupported interpretations, explanations and "mysteries of the growth process" (Galor, 2005a, p. 220).*

In the distorted and appropriately manipulated presentation of data shown in Figure 1 we can see clearly the non-monotonic growth of population and of the GDP/cap. After the apparent long stagnation, we see a sudden takeoff to a new regime of growth. Galor made no attempt to analyse data, which is surprising because their analysis is trivially simple (Nielsen, 2014). The manipulated data appear to support the concept of stagnation and takeoffs described usually as the escape from the Malthusian trap.

In contrast, the accurate display of precisely the same data suggests entirely different interpretation. General features presented in Figure 1 are still maintained but now mathematical analysis of these data shows that the GDP/cap and the size of the population were increasing *monotonically* (Nielsen, 2014; 2015a; 2016a; 2016d; 2016e; 2016f; 2016g; 2016h). There were no sudden takeoffs from stagnation to growth because there was no stagnation and because the acceleration was gradual along the entire range of these distributions. The gradient and the growth rate of the GDP/cap distribution were changing monotonically without any discontinuity, which could be claimed as a takeoff (Nielsen, 2015a, 2016a, 2016h).

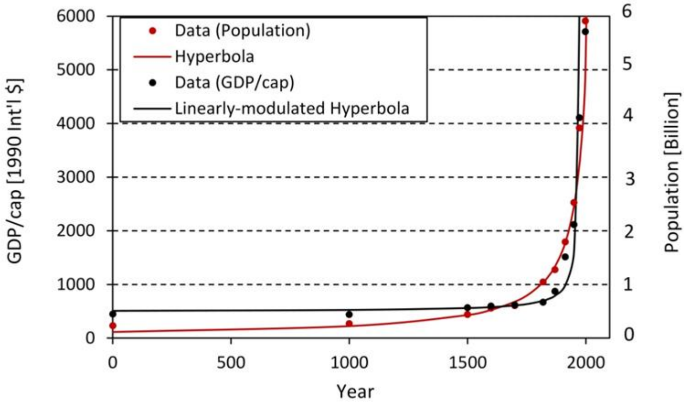

Figure 2. *Precisely the same data (Maddison, 2001) as used in Figure 1but now displayed accurately and analysed. They follow monotonically-increasing distributions, which cannot be divided into distinctively-different components (Nielsen, 2014, 2015a, 2016a, 2016d, 2016e, 2016f, 2016g, 2016h).*

Even though the GDP/cap distribution seems to suggest a sudden increase, this feature is just an illusion, which is dispelled by the mathematical analysis of data or even by using semi logarithmic scales of reference (Nielsen, 2015a, 2016g). The GDP/cap is the ratio of two distributions, the distribution describing the growth of the GDP and the distribution describing the growth of population. Both were increasing hyperbolically and monotonically (Nielsen, 2015a, 2016a, 2016d). The displayed features (slow growth over a long time and fast growth over a short time) represent nothing more than mathematical properties of monotonically-increasing hyperbolic distributions. They are not the unique properties of economic growth but economic growth happens to be hyperbolic.

It is impossible to locate a transition from the slow to fast growth for hyperbolic distributions (Nielsen, 2014) because such a transition does not exist. The GDP/cap distributions are simply the linearly-modulated and monotonically-increasing hyperbolic distributions (Nielsen, 2015a).

The distorted diagram used by Galor to support his erroneous concept of the great divergence is presented in Figure 3. This distorted presentation of Maddison's data was reproduced from Galor's publication (Galor, 2005a, p. 175). It shows that over a long time there was hardly any difference in the economic growth for various regions. However, from around the time of the Industrial Revolution, 1760-1840 (Floud & McCloskey, 1994), there was a sudden takeoff and the economic growth diverged into distinctly different trajectories.

We have already demonstrated that there were no takeoffs in the growth of the GDP and GDP/cap (Nielsen, 2015a, 2016a, 2016e, 2016g) and consequently there was also no differential timing of takeoffs claimed by Galor in his Unified Growth Theory (Galor, 2005a, 2011). We have also demonstrated that there were no takeoffs in the growth of the world population in the past 12,000 years (Nielsen, 2016b) and in the growth of regional populations (Nielsen, 2016d). The incorrectly-claimed takeoffs by Galor represent just the natural continuations of hyperbolic growth. Analysis of data shows that at the time of the supposed takeoffs, and in clear contradiction of the Unified Growth Theory, economic growth in various regions was either continuing to increase along undisturbed hyperbolic trajectories or started to be diverted to *slower* trajectories.

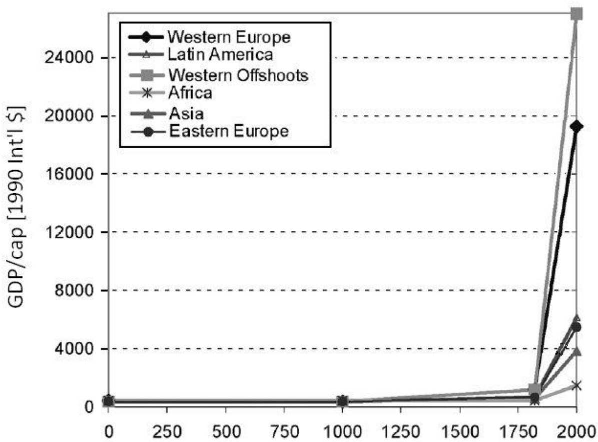

Figure 3. *A typical distorted presentation of Maddison's data (Maddison, 2001) used by Galor to support his concepts of takeoffs and of the great divergence (Galor, 2005a, p. 175).*

Now we shall show that there was no great divergence in the income per capita. We shall show again that the Unified Growth Theory is scientifically unacceptable. It does not describe the mechanism of economic growth. It describes phantom features constructed by the manipulation of data.

We shall show that the great divergence never happened. However, we shall also *explain* how Galor constructed his great divergence. We shall show how the great divergence can be constructed by a distorted presentation of any distributions, which increase slowly over a long time and fast over a short time. They do not have to be distributions describing economic growth.

Analysis of the early data of Maddison

We shall first investigate *precisely the same data* (Maddison, 2001) as used by Galor (2005a; 2011) during the formulation of his Unified Growth Theory and we shall show that they do not support the concept of the great divergence. Results of mathematical analysis of these data are shown in Figures 4-9. Parameters of the fitted distributions have been listed earlier (Nielsen, 2016g). The fitted curves are the linearly-modulated hyperbolic distributions (Nielsen, 2015a) obtained by dividing hyperbolic distributions fitting the corresponding GDP and population data (Nielsen, 2016a; 2016d). All GDP/cap values are in 1990 International Geary-Khamis dollars.

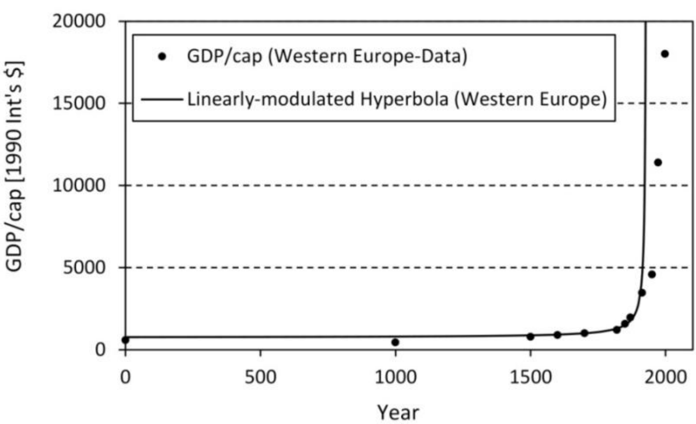

Figure 4. *Growth of income per capita, i.e. Gross Domestic Product per capita (GDP/cap), in Western Europe (Maddison, 2001; Nielsen, 2016g). From around 1913, economic growth in Western Europe started to depart from the historical linearly-modulated hyperbolic distribution. However, it continued to increase close to the historically-predicted trajectory.*

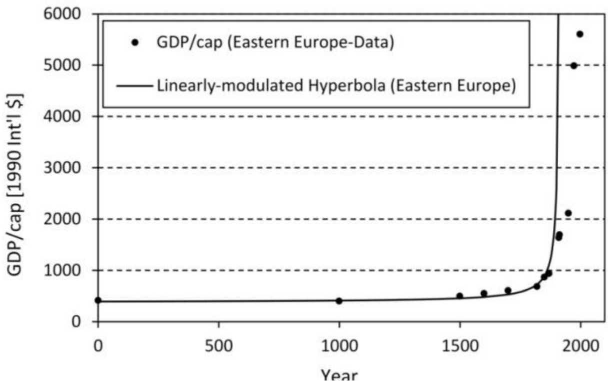

Figure 5. *Growth of income per capita in Eastern Europe (Maddison, 2001; Nielsen, 2016g). From around 1870, economic growth in Eastern Europe started to depart from the historical linearly-modulated hyperbolic distribution. However, it continued to increase close to the historically-predicted trajectory. The growth was not diverted to a distinctly different and gently-increasing trajectory as claimed by Galor (2005a; 2011; cf Figure 3).*

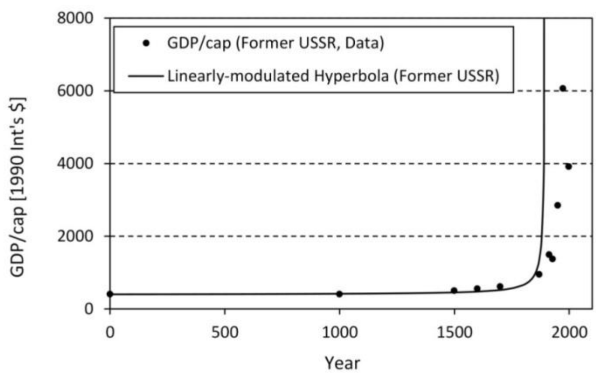

Figure 6. *Growth of income per capita in countries of the former USSR (Maddison, 2001; Nielsen, 2016g). From around 1870, economic growth in the former USSR started to depart from the historical linearly-modulated hyperbolic distribution. However, it continued to increase close to the historically-predicted trajectory.*

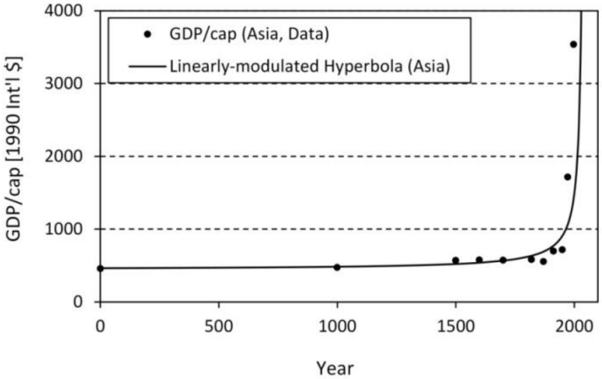

Figure 7. *Growth of income per capita in Asia (Maddison, 2001; Nielsen, 2016g). The data follow closely the linearly-modulated hyperbolic distribution. There was no divergence to a distinctly different and gently-increasing trajectory as claimed by Galor (2005a; 2011;cf Figure 3).*

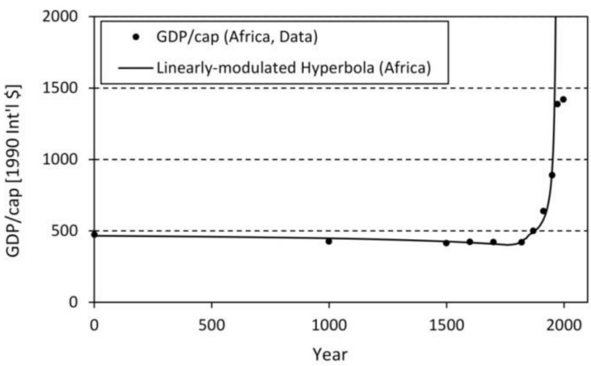

Figure 8. *Growth of income per capita in Africa (Maddison, 2001; Nielsen, 2016g). The data follow closely the linearly-modulated hyperbolic distributions. There was no divergence to a distinctly different and gently-increasing trajectory as claimed by Galor (2005a; 2011;cf Figure 3).*

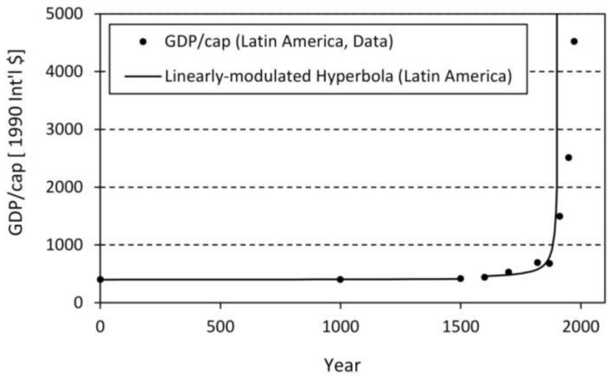

Figure 9. *Growth of income per capita in Latin America (Maddison, 2001; Nielsen, 2016g). From around 1913, economic growth in Latin America started to depart from the historical linearly-modulated hyperbolic distribution. However, it continued to increase close to the historically-predicted trajectory. The growth was not diverted to a distinctly different and gently-increasing trajectory as claimed by Galor (2005a; 2011, cf Figure 3).*

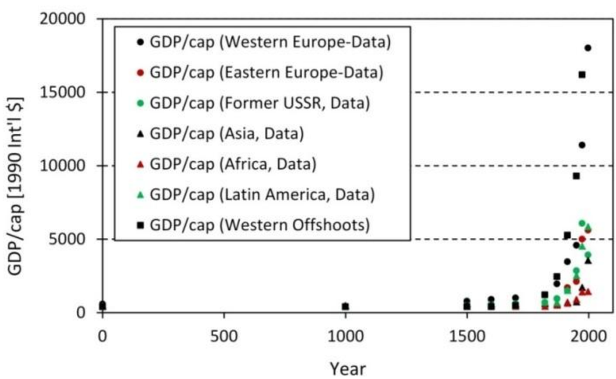

Figure 10. *Growth of income per capita in all regions, including Western offshoots (Maddison, 2001). They are all increasing in approximately the same direction. There is no divergence to distinctly different trajectories.*

The data for Western Offshoots were not analysed because of their poor quality, but they are displayed in Figure 10. Their economic growth is similar to the growth in Western Europe in the sense that they are clearly ahead of other regions. However, distributions presented in Figures 4-9 show that economic growth in all regions follows similar trajectories. The difference between regions is not in their divergence to distinctly different trajectories as claimed incorrectly by Galor but in their *levels* of economic development.

Distributions presented in Figures 4-9 are clearly different than the distorted distributions constructed by Galor and presented in Figure 3. In Galor's distorted presentation of data there is a cluster of regions (Eastern Europe, Asia, Africa and Latin America) whose economic growth follows distinctly different trajectories than the growth in Western Europe. This information is incorrect because the analysis of precisely the same data shows clearly that all distributions are similar, including the distribution representing the economic growth in Africa. They are all following similar trajectories with a common tendency to increase nearly vertically and close to the historical linearly-modulated hyperbolic trajectories.

The common characteristic feature of all these empirical distributions shown in Figures 4-9 is that they have changed gradually from being nearly horizontal to nearly vertical. We shall show later that when such distributions become nearly vertical it is easy to distort them and construct the great divergence, and it does not matter whether they follow the fitted linearly-modulated hyperbolic distributions or not.

The contrast between Maddison's data and their distorted image constructed by Galor is particularly clear if we compare Figure 3 with Figure 8. In Figure 3, the data for Africa follow a gently-increasing trajectory after around 1800, i.e. a trajectory characterised by a small gradient. The correct display of *the same data* presented in Figure 8 shows diametrically opposite features: the data for Africa follow a *steep* trajectory, i.e. the trajectory characterised by a large gradient. This trajectory is approximately vertical.

In Galor's distorted presentation of data the trajectory for Africa after around 1800 is distinctly different than the trajectory for Western Europe. However, precisely the same data displayed in Figures 4 and 8 demonstrate that the trajectories for Africa and Western Europe are similar. The only difference is that Africa is further behind in its level of development.

In Galor's distorted presentation of data, economic growth in Eastern Europe, Asia and Latin America follows also gently increasing trajectories after around 1800, similar to the trajectory for Africa. However, precisely the same data displayed properly in Figures 5, 7, 8 and 9 show that they all follow approximately vertical trajectories in much the same way as the data for Western Europe. The only difference is again that Western Europe is further ahead but it is further ahead on the virtually the same trajectory.

With such distorted presentation of data, it is not surprising that Galor discovered so many "mind-boggling" and "perplexing" "mysteries of the growth process" (Galor, 2005a, pp. 177, 220), mysteries representing phantom features created by the manipulation of data.

In contrast with his distorted presentation of data, the gradient of all empirical trajectories in this section of time is large. They all increase approximately vertically. Such a growth cannot be explained by claiming that larger size of population demands larger GDP. What we have here is the increasing GDP *per person*. It is a growth that reflects our surprisingly fast-increasing *demands*.

Galor's theory conveys dangerously incorrect information. According to his distorted presentation of data shown in Figure 3, income per capita in certain regions (Eastern Europe, Asia, Africa and Latin America) is following gently increasing trajectories after around 1800. Such trajectories are relatively safe. However, the correct presentation of precisely the same data shows that in *all* regions income per capita is following the dangerously fast-increasing trajectories. The data show that there is now a critical urgency to regulate economic growth but Galor's theory suggests that there is no danger.

According to his erroneous theory, after a long epoch of stagnation we have escaped the tyranny of the Malthusian trap and now we can enjoy the sustained growth regime. Furthermore, according to his erroneous concept of the great divergence, economic growth in most regions diverged to the generally safe trajectories. However, according to the precisely the same data, all regions are now following dangerously fast-increasing trajectories and for all of them, without exception, economic crisis seems to be strongly probable.

Analysis of the latest data of Maddison

Data published by Maddison in 2010 show even more clearly that there was no divergence in the economic growth. These data were available to Galor before the publications of his book (Galor, 2011) but unfortunately, they were not analysed. Had Galor analysed these data he would have soon discovered many interesting features characterising economic growth, features, which are repeatedly in contradiction with his Unified Growth Theory (Galor, 2005a; 2011).

Results of analysis of these new data (Maddison, 2010) are shown in Figures 11-16. Their combined display is presented in Figure 17.

The mystery of Galor's "mind-boggling" and "perplexing phenomenon of the Great Divergence" (Galor, 2005a, pp. 177, 220) has now been solved – there is no mystery. This mystery and all other of his mysteries were created by the manipulation of data. In Galor's publications (2005b; 2007; 2008a; 2008b; 2008c; 2010; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008) data are repeatedly manipulated and presented using distorted diagrams.

The common characteristic feature of Maddison's data describing the growth of income per capita (Maddison, 2001; 2010) in various regions is again that their nearly horizontal trajectories changed *gradually* into nearly vertical trajectories. They have *never diverged* into distinctly different trajectories as claimed by Galor (see Figure 3).

Economic growth in all regions is now following new trajectories but all of them continue to increase close to the historical, linearly-modulated hyperbolic trajectories, which escape to infinity at a fixed time. In contrast with Galor's interpretation based on his erroneous concept of the great divergence, all new trajectories are critically fast. They do not increase to infinity at a fixed time but they pose virtually the same danger as the historical, linearly-modulated hyperbolic trajectories because they are close to the trajectories, which increase to infinity at a fixed time.

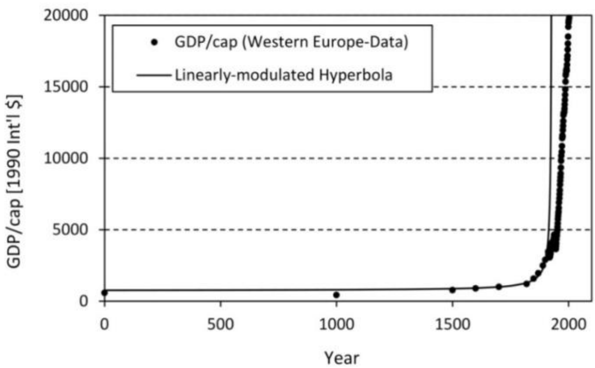

Figure 11. *Growth of income per capita in Western Europe (Maddison, 2010; Nielsen, 2016g). Between 1900 and 1913, economic growth in Western Europe started to depart from the historical linearly-modulated hyperbolic distribution. However, it continues to increase close to the historically-predicted trajectory.*

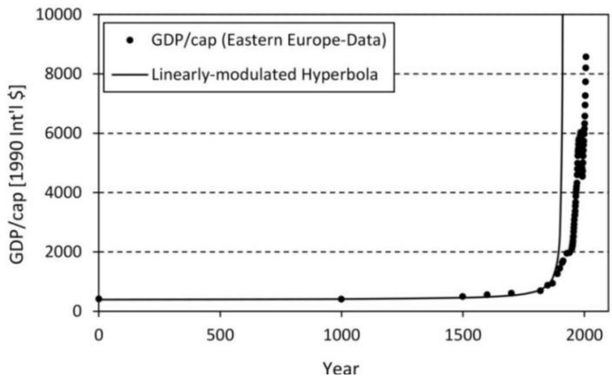

Figure 12. *Growth of income per capita in Eastern Europe (Maddison, 2010; Nielsen, 2016g). From around 1850 economic growth in Eastern Europe started to depart from the historical linearly-modulated hyperbolic distribution. However, it continues to increase close to the historically-predicted trajectory. The growth was not diverted to a distinctly different and gently-increasing trajectory as claimed by Galor (2005a; 2011; cf Figure 3).*

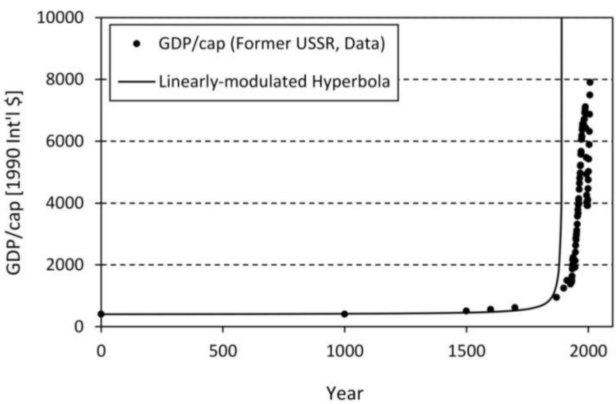

Figure 13. *Growth of income per capita in countries of the former USSR (Maddison, 2010; Nielsen, 2016g). Close to around 1870 economic growth in countries of the former USSR started to depart from the historical linearly-modulated hyperbolic distribution. However, it continues to increase close to the historically-predicted trajectory.*

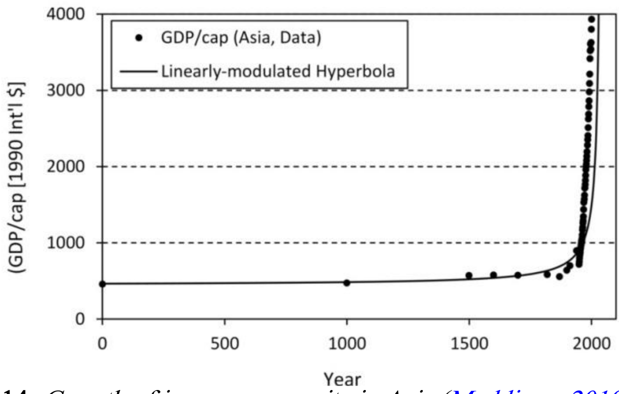

Figure 14. *Growth of income per capita in Asia (Maddison, 2010; Nielsen, 2016g). After a brief decline between 1940 and 1950, the growth of income per capita in Asia was diverted to a slightly faster trajectory. However, it continues to increase close to the historically-predicted linearly-modulated hyperbolic distribution. The growth was not diverted to a distinctly different and gently-increasing trajectory as claimed by Galor (2005a; 2011; cf Figure 3).*

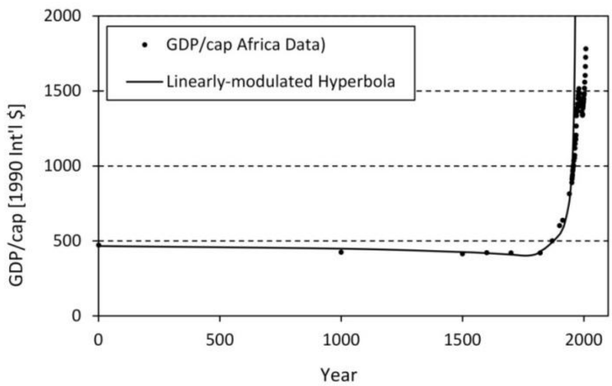

Figure 15. *Growth of income per capita in Africa (Maddison, 2010; Nielsen, 2016g). In clear contradiction of Galor's claim supported by his distorted presentation of Maddison's data, the growth of income per capita did not diverge to a slowly-increasing trajectory but continues to increase along a nearly vertical trend close to the historical linearly-modulated hyperbolic distribution (cf Figure 3).*

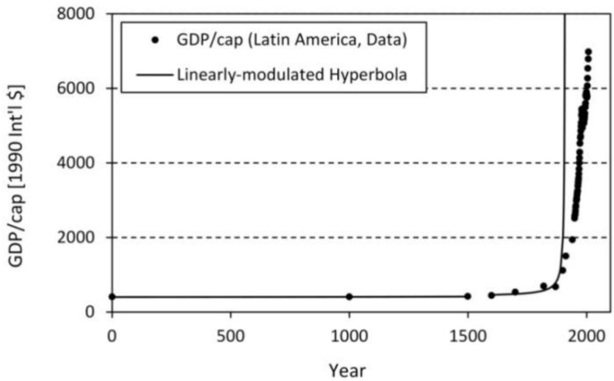

Figure 16. *Growth of income per capita in Latin America (Maddison, 2010; Nielsen, 2016g). In clear contradiction of Galor's claim supported by his distorted presentation of Maddison's data, growth of income per capita continued to increase along a nearly vertical trajectory close to the historical linearly-modulated hyperbolic distribution. The growth was not diverted to a distinctly different and gently-increasing trajectory as claimed by Galor (2005a; 2011; cf Figure 3).*

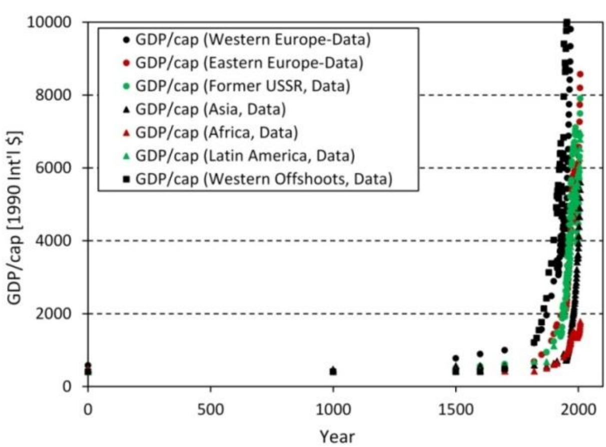

Figure 17. *Growth of income per capita in all regions, including Western offshoots (Maddison, 2010). Even without carrying out mathematical analysis of data it is clear that they all follow similar, nearly-vertical trajectories. The mystery of the "mind-boggling" and "perplexing phenomenon of the Great Divergence" (Galor, 2005a, pp. 177, 220) has now been explained – there is no mystery. The great divergence never happened. This mystery, as well as all his other "unresolved mysteries about the growth process" (Galor, 2005a, p. 220) have been created by the mind-boggling, perplexing and self-misleading manipulation of data.*

In Galor's distorted presentation of data shown in Figure 3, economic growth in various regions follows similar trajectories for a long time and then diverges to distinctly different trajectories. In the correct and undistorted presentation of data shown in Figure 17, economic growth in various regions follows similar trajectories *all the time*. Some regions are slower in their economic development but they all race in the same direction and along virtually the same trajectory. They do not fan out into distinctly different directions as claimed by Galor.

We do not have to explain the mechanism of the great divergence because the great divergence never happened. It is a feature created by the distorted presentation of data. If we want to explain the currently observed differences in the economic growth we should not be misguided by the Unified Growth Theory and we should not attempt to explain why different regions follow distinctly different trajectories, because they do not follow distinctly different trajectories. We should rather try to explain why different regions follow *similar* trajectories and why for some regions economic growth is faster while for other regions it is slower.

Geometric distortions

We shall now explain *how* Galor constructed his "unresolved mysteries about the growth process" (Galor, 2005a, p. 220): (1) his "mind-boggling" and "perplexing phenomenon of the Great Divergence" (Galor, 2005a, pp. 177, 220) and (2) his equally mind-boggling but fictitious takeoffs from the supposed but non-existent stagnation to growth. To demonstrate how such mysteries are created, we can take any family of distributions, which change slowly over a large range of independent variable and fast over its short range. We can use hyperbolic distributions, linearly-modulated hyperbolic distributions, a set of empirical distributions such as shown in Figures 10 and 17, or any other hyperbolic-like distributions. By their simple manipulation we can easily create Galor's "mysteries about the growth process" (Galor, 2005a, p. 220) but they will not be unresolved mysteries. They will not even be mysteries because we shall demonstrate and explain their origin. We shall demonstrate that these supposed mysteries do not represent unique properties of economic growth but the introduced by us disfigurations of hyperbolic-like distributions.

For our demonstration we have chosen three, closely-related linearly-modulated hyperbolic distributions shown in Figure 18. Like the historical income per capita distributions, each of these arbitrary distributions is represented by a ratio of two hyperbolic distributions. However, they have absolutely nothing to do with economic growth. They are purely mathematical functions $f(x)$, $g(x)$ and $h(x)$ where x is an arbitrary independent variable. This variable could be time but it could be also anything else. The common feature of these distributions is that they start from approximately the same value at $x=0$, they increase *monotonically* (they are not characterised by sudden takeoffs at any time) and they increase to infinity within a small range of x values. *They do not diverge.*

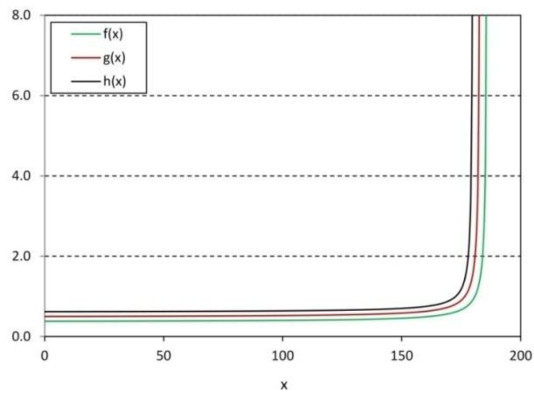

Figure 18. *Three arbitrarily-chosen, linearly-modulated, hyperbolic distributions,* $f(x)$*,* $g(x)$ *and* $h(x)$*. They increase* monotonically *from approximately the same value at* $x=0$ *to infinity within approximately the same time. They do not diverge.*

However, if we follow Galor's example we can use these *non-divergent* and *monotonically*-increasing distributions and *construct a new set of diverging distributions*, which will be also characterised by clear takeoffs. All we have to do is to select a few strategically-located points at certain constant *x*-values and join them by straight lines. This is precisely what Galor was doing repeatedly during the formulation of his Unified Growth Theory (Galor, 2005a; 2011) and in his other publications (Galor, 2005b; 2007; 2008a; 2008b; 2008c; 2010; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008).

We have selected three values of independent variable, $x=0$, $x=150$ and $x=179.6$, and by following Galor's example, we have connected the corresponding values of $f(x)$, $g(x)$ and $h(x)$ by straight lines. We have now *constructed* typical distributions used by Galor to formulate his Unified Growth Theory. We have also *constructed* the great divergence and the takeoffs. Results are shown in Figure 19.

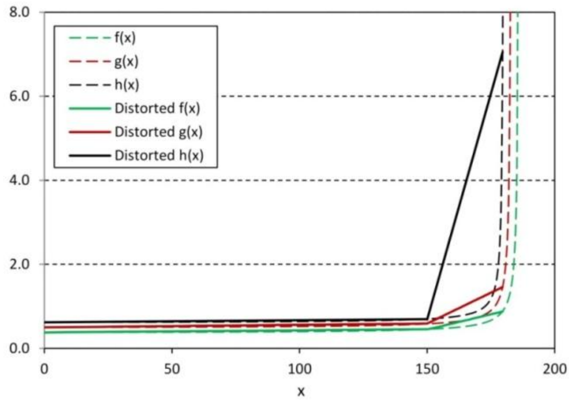

Figure 19. *This figure explains how the "mind-boggling" and "perplexing phenomenon of the Great Divergence" (Galor, 2005a, pp. 177, 220) was invented by Galor and how he created his otherwise non-existing takeoffs from the non-existing stagnation to growth. By following his approach to research, the corresponding values of the purely-mathematical functions* $f(x)$, $g(x)$ *and* $h(x)$ *at* $x = 0$, $x = 150$ *and* $x = 179.6$ *were joined by straight lines. The monotonically-increasing distributions are now replaced by distorted diagrams in much the same way as Maddison's data were replaced by Galor by his distorted diagrams. We have constructed the meaningless "mind-boggling" and "perplexing phenomenon of the Great Divergence" (Galor, 2005a, pp. 177, 220) preceded by the equally meaningless takeoffs at* $x = 150$.

It would be incorrect to claim that our constructed distributions shown in Figure 19 represent the original distributions, which were shown in Figure 18, but Galor repeatedly and incorrectly uses his distorted diagrams as representing Maddison's data. His repeatedly used diagrams are the *misrepresentations of data* and his conclusions based on such diagrams or on quoting some isolated numbers selected from hyperbolic distributions are scientifically unacceptable and strongly misleading.

By using the constructed great divergence and the takeoffs shown in Figure 19 and by constructing more of such diagrams we could now create a unified growth theory describing properties of the distorted diagrams and insist that they represent mathematical properties of $f(x)$, $g(x)$ and $h(x)$ functions or the properties of other similar distributions. However, it would be naive for us to expect that people familiar with mathematics would be impressed by our scholarly performance and by the mysteries we have created. It would be naïve to expect that they would accept our explanations of the claimed mathematical properties of hyperbolic-like distributions, and yet Galor expects that economists will accept his distorted representations of Maddison's data and his explanations of economic growth based on such repeatedly distorted presentations of data reinforced by the numerous quotations of well-selected and isolated numbers, which are supposed to represent a reliable empirical confirmation of his theory.

Like Galor, we could claim the existence of takeoffs from stagnation to growth for our mathematical, monotonically-increasing functions $f(x)$, $g(x)$ and $h(x)$. We could try to explain these phantom takeoffs by some fanciful mechanisms, but such explanations would be unacceptable because the original functions increase monotonically. They are not characterised by sudden takeoffs. These takeoffs do not exist. We have created them by distorting the original functions in much the same way as Galor created them by distorting Maddison's data.

Like Galor, we could also claim the existence of the great divergence and try to explain it by some complicated mechanisms but again our claim and our explanations would be unacceptable because the original functions do not diverge. We have created the great divergence, which does not characterise the original functions but only their distorted representations. Like Galor, we could claim the existence of the "unresolved mysteries" (Galor, 2005a, p. 220) about mathematical functions but the only audience we could hope to impress would be people who are not familiar with mathematics but it is also possible that even people unfamiliar with mathematics would soon notice that what we are doing is just clever or maybe even not so clever sophism.

Conclusions based on the distorted representations of mathematical distributions $f(x)$, $g(x)$ and $h(x)$ can be obviously rejected. Likewise, conclusions based on Galor's distorted representations of Maddison's data can be and even should be rejected. Galor presents many curious and seemingly logical stories about economic growth but his stories are either repeatedly contradicted by data (Nielsen, 2014; 2015a; 2016a; 2016b; 2016d; 2016e; 2016f; 2016g; 2015h) or have no convincing confirmation in data. They have to be accepted largely by faith. Stories of fiction can be also attractive, logical and convincing but they will remain stories of fiction.

It would be incorrect to claim that the distorted diagrams presented in Figure 19 represent the mathematical distributions $f(x)$, $g(x)$ and $h(x)$. Likewise, it would be incorrect to claim that the distorted diagrams used repeatedly by Galor in his Unified Growth Theory and in his other publications represent Maddison's data.

It would be incorrect to claim that the distorted diagrams presented in Figure 19 describe the mathematical functions $f(x)$, $g(x)$ and $h(x)$. Likewise, it would be incorrect to claim that the distorted diagrams presented by Galor in his Unified Growth Theory and in his many other publications describe economic growth. They describe the world of fiction.

It could be hardly expected that explanations of the properties of mathematical functions $f(x)$, $g(x)$ and $h(x)$ based on their distorted representations shown in Figure 19 could be ever accepted by people familiar with mathematics. Likewise, it can be hardly expected that explanations of economic growth based on such distorted presentations of data as used by Galor in his Unified Growth Theory and in his other publications can be accepted by the scientific community.

Discussion and conclusions

We have analysed Maddison's data (Maddison, 2001; 2010) and we have demonstrated that the great divergence claimed by Galor (2005a, 2011) and shown in Figure 3 never happened. Various regions are now on different *levels* of development but their economic growth *did not diverge* into distinctly different trajectories as claimed by Galor (see Figure 3). Their income per capita increases along similar, approximately vertical trajectories.

The disagreement between Galor's claim and the data can be demonstrated using the early Maddison's data (Maddison, 2001), which Galor used in their habitually distorted presentations during the formulation of his Unified Growth Theory (Galor, 2005a, 2010). However, the disagreement between his claims and the data becomes even more pronounced if we display the latest data of Maddison (2010), which were available to Galor before the publication of his book (Galor, 2011).

The data do not even have to be analysed mathematically to show that they contradict Galor's claim of the existence of the great divergence but their mathematical analysis is helpful. Galor's claims expressed in his Unified Growth Theory and in his other similar publications are based on his failure to adhere to the fundamental and indispensable principles of scientific investigation, which require that data should be rigorously analysed, that conclusions should not be based on impressions and that data should not be manipulated to support preconceived ideas. His theory, his claims and his interpretations are scientifically unacceptable.

"The mind-boggling phenomenon of the Great Divergence in income per capita across regions of the world in the past two centuries, that accompanied the take-off from an epoch of stagnation to a state of sustained economic growth, presents additional unresolved mysteries about the growth process" (Galor, 2005a, p. 220). It is interesting how a single sentence can contain so much misinformation.

His mysteries have now been solved: he has created them by the manipulation of data.

The great divergence never happened and neither did the takeoffs from Malthusian stagnation to growth (Nielsen, 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2015h). Within the range of analysable data there was no stagnation and no transition from stagnation to growth. Features described by Galor as takeoffs are not takeoffs but the natural continuations of monotonically-increasing hyperbolic distributions describing the growth of the GDP or population, or the natural continuations of monotonically-increasing linearly-modulated distributions describing the growth of the GDP/cap. Hyperbolic distributions or linearly-modulated hyperbolic distributions are slow over a long time and fast over a short time but they do not change suddenly from slow to fast at any time. They increase monotonically all the time.

Hyperbolic growth excludes the interpretations revolving around the concept of Malthusian stagnation and around takeoffs from stagnation to growth described usually as the escape from the Malthusian trap. The evidence contradicting such interpretations is overwhelming. It is remarkable that so many independent studies are in such perfect agreement: Maddison's data (Maddison, 2001, 2010) and their analysis (Nielsen, 2014; 2015a; 2016a; 2016d; 2016e; 2016f; 2016g; 2015h); the estimates of the size of human population not only during the AD era but also during the BC era (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1967, 1974, 1977; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994) and their analysis (e.g. Kremer, 1993; Nielsen, 2016b; Kapitza, 2006); the discovery made by von Foerster, Mora & Amiot (1960) and similar identifications of hyperbolic growth by Podlazov (2002), Shklovskii (1962; 2002) and von Hoerner (1975).

According to Galor, the "differential timing of the take-off from stagnation to growth across countries, and the corresponding variations in the timing of the demographic transition, led to a great divergence in income per capita as well as population growth" (Galor, 2005a, p. 218). This is a good example how fiction can be created even in science. Non-existent takeoffs have been constructed by distorted presentations of data. These non-existent takeoffs were then used to explain the non-existent differential timing of takeoffs, and now the same phantom takeoffs are used to explain the origin of the non-existent great divergence constructed by the manipulation of data.

Galor wonders about "the underlying driving forces that triggered the recent transition between these regimes and the associated phenomenon of the Great Divergence in income per capita across countries" (Galor, 2005a, pp. 174, 219). While it is interesting to study reasons for differences in the *level* of economic growth of various regions and countries, there is no need to wonder about the underlying driving forces of the great divergence because the great divergence never happened.

Galor claims that "In the course of the 'Great Divergence' the ratio of GDP per capita between the richest region and the poorest region has widened considerably from a modest 3 : 1 ratio in 1820, to a 5 : 1 ratio in 1870, a 9 : 1 ratio in 1913, a 15 : 1 ratio in 1950, and a 18 : 1 ratio in 2001." (Galor, 2005a, p. 174). All these ratios are probably correct but the conclusion is incorrect because there was no great divergence. Economic growth in various regions is on different *levels* of development but it follows the virtually the same, *non-diverging,* trajectories.

This is a good example of being guided by impressions and of using them to draw hasty conclusions. Data have to be rigorously analysed. Using isolated numbers, as done repeatedly by Galor, is likely to lead to incorrect interpretations particularly if such a use of isolated numbers is combined with the repeatedly distorted presentation of data, such as shown in Figures1 and 3. Taking shortcuts and using them to draw hasty conclusions based usually on preconceived ideas and on wished-for interpretations does not represent scientific process of investigation. The ratios listed by Galor do not prove the existence of the great divergence. We have already demonstrated that the great divergence never happened. The listed ratios represent nothing more than hyperbolic growth and different levels of development *along virtually identical trajectories*.

Current economic growth in various regions and countries is at different *levels* of development. For countries characterised by high human development, income per capita can be as high as tens of thousands of dollars but for countries characterised by low human development it can be about one hundred times lower (Nielsen, 2006). However, Maddison's data show that economic growth in all regions, without exception, is developing along *virtually the same trajectories.*

Galor's interpretation of economic growth is potentially dangerous because it creates a false sense of security. He shows that gradients of the current economic-growth trajectories are in general small and consequently the imminent economic crisis is unlikely (see Figure 3).

However, data convey totally different information. Economic growth in *all* regions is now increasing rapidly along virtually vertical trajectories (see Figures 4-17). They resemble the historical linearly-modulated trajectories, which increase to infinity at a fixed time. For such trajectories, economic crisis can be expected because the growth has to be supported by excessively large per annum increase in the GDP per capita. The created stress can be too high to be manageable over a long time. There is also a danger of reaching quickly natural limits to growth.

Warning signs can be already seen in Eastern Europe, in countries of the former USSR and in Africa (Figures 12, 13, 15). Their growth of income per capita suffered reversals but after a certain time it managed to recover and follow again the nearly vertical trajectories. Certain degree of instability can be also observed in Latin America (Figure 16).

The preferred option would be to follow now gently-increasing trajectories but all regions, without exception, appear to be caught up in the general frenzy to increase rapidly their *per capita* economic growth. When they are temporarily left behind they soon resume their hazardous race. Current trajectories do not increase to infinity at a fixed time but they increase to infinity in a short time, which is hardly a consolation.

All these important warning signs are not even noticed in the Unified Growth Theory. Unified Growth Theory appears to suggest a prosperous future after an ages-long epoch of a hypothetical stagnation but the data show that the future of economic growth is approaching rapidly levels of unsustainability. It has been shown that the world economic growth follows unsustainable trajectory (Nielsen, 2015b). However, the analysis of Maddison's data presented here suggests that this is a common danger shared by *all* regions. There is not a single region, whose economic growth diverged to a safer trajectory.

The two opposite interpretations of economic growth have also essential impact on research activities. In order to explain Galor's great divergence we would have to explain why there was a transition to distinctly different trajectories of economic growth. Such attempts would be a waste financial and human resources and a waste of time because the great divergence never happened. What we have to explain is why different regions follow *virtually the same* trajectories and why they follow such potentially-hazardous, fast-increasing trajectories. Why there is such a strong desire to increase the GDP *per capita* so quickly everywhere and how to control these dangerous tendencies.

Galor claims that the "transitions from a Malthusian epoch to a state of sustained economic growth and the emergence of the Great Divergence have shaped the current growth process in the world economy" (Galor, 2005a, p. 221). They did not because there was no "emergence of the Great Divergence." Galor describes phantom features he created by his manipulation of data. These phantom features could not have shaped the past growth and they do not shape the current growth because they did not and do not exist. Galor describes the world of fiction and events that never happened. He then uses these non-existing phenomena to weave his theory around them.

Transitions from the Malthusian epoch of stagnation to a state of sustained economic growth never happened because there was no stagnation. Economic growth was sustained in the past because it followed steadily-increasing hyperbolic trajectories. Takeoffs, which are supposed to represent the claimed transitions from stagnation to growth, never happened (Nielsen, 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2015h).

Galor's claims are based on distorted presentations of data and generally on repeatedly violating the fundamental principles of scientific investigation. They are based on impressions rather than on the rigorous scientific analysis of empirical evidence.

Galor claims that the "unified growth theory sheds light on the perplexing phenomenon of the Great Divergence in income per capita across regions of the world in the past two centuries" (Galor, 2005a, p. 177). If it does, then his theory is a fiction because the perplexing phenomenon of the great divergence never happened.

Why did we devote so much time on the discussion of Galor's Unified Growth Theory (Nielsen, 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2015h)? The answer is simple. As for Isaak Newton, the aim of any scientific investigation is to discover the truth. It is not a person dislike. Science looks for correct interpretations but Galor's theory is so obviously incorrect that it attracted immediate attention.

However, there is also another important reason: Galor's Unified Growth Theory is not only incorrect but also dangerously incorrect because it diverts attention from the urgent need to monitor, control and regulate the current economic growth. It would be unwise to accept his theory and his explanations because his incorrect explanations of the historical economic growth are linked strongly with the current economic growth, which affects our future.

Galor claims that after a long epoch of stagnation we are now in the regime of sustained economic growth. His theory also strongly suggests that the current economic growth is not only sustained but also sustainable because in general it follows slowly-increasing trajectories (see Figure 3). The future appears to be safe and secure.

However, precisely the same data, which he used during the formulation of his theory, show that the opposite is true. It was in the past that the economic growth was safe and secure but now it follows strongly hazardous trajectories. Recent analysis of the world economic growth also indicates that its future is insecure (Nielsen, 2015b), which is hardly surprising because our current combined ecological footprint is already significantly higher than the ecological capacity (WWF, 2010).

Why did Galor manipulate data? Why did he repeatedly present distorted diagrams to support his preconceived ideas? Why did he quote isolated and well-chosen but otherwise meaningless numbers to support his arguments? Why did he create such an elaborate work of fiction?

If we assume that he did not do it all on purpose, then a possible explanation is that he did not know how to analyse data. However, this explanation is unconvincing because he appears to be familiar with mathematics. Anyone familiar with mathematics can see quickly that plots of Maddison's data display characteristic features of hyperbolic distribution. Anyone familiar with mathematics knows also that the analysis of hyperbolic distributions is trivially simple (Nielsen, 2014). However, equally surprising is why his publications escaped the scrutiny of the peer-review system.

The most plausible explanation is probably that he was blinded by prejudice. It is what psychologists describe as the cascade behaviour, information cascade, informational avalanche, illusion of truth, illusory truth, illusion of familiarity, running with the pack, following the crowd, herding behaviour, bandwagons and path depending choice (Anderson & Holt, 1997; Begg, Anas &

Farinacci, 1992; Bikhchandani, Hirshleifer & Welch, 1992; De Vany & Lee, 2008; De Vany & Walls, 1999; Easley & Kleinberg, 2010; Grebe, Schmid & Stiehler, 2008; Ondrias, 1999; Parks & Tooth, 2006; Ramsey, Raafat, Chater & Frith, 2009; Walden & Browne, 2003). It is the fear of being different, of taking risks, of sticking the neck out, of claiming something, which is not commonly accepted.

In certain areas of intellectual activities, this problem creates nearly insurmountable obstacles. In the demographic and economic research this phenomenon is demonstrated by the reluctance to accept the compelling contradicting evidence simply because many demographers or economists would not agree with the contradicting evidence. It is safer to follow the crowd and run with the pack. Tradition is stronger than science and only an outsider who has not been blinded by prejudice and who is not afraid of being rejected by the crowd might dare to show that the accepted doctrines are incorrect. He or she is then risking to be ridiculed and rejected but science is a self-correcting discipline so sooner or later such resistance to accept the overwhelming empirical evidence will have to be overcome, but it would be better for science and scientists if the required change in the paradigm is accepted sooner rather than later.

We now have a large body of data (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1967, 1974, 1977; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Maddison, 2001, 2010; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994), which can be used to improve our understanding of the economic growth and of the growth of human population. Correct understanding of these two processes might have essential impact on our future.

The recent mathematical analysis of data (Nielsen, 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2015h) reveals many interesting features, which call for further investigation. The past economic growth and the growth of human population were hyperbolic. Within the range of analysable data, which for the growth of human population extends down to 10,000 BC, there was no Malthusian stagnation. Hyperbolic growth was slow but remarkably steady. There were no transitions from stagnation to growth because there was no stagnation. There was no escape from the Malthusian trap in the economic growth or in the growth of population because there was no trap. There were no takeoffs from stagnation to growth claimed by Galor (2005a, 2011). There was no differential timing of takeoffs, claimed also by Galor, because there were no takeoffs.

We have demonstrated (Nielsen, 2016h) that there was no "sudden spurt in growth rates of output per capita" (Galor, 2005a, p. 220). Contrary to the similar claim made by Galor, there was also no sudden spurt in the growth rate of human population in the past 12,000 years (Nielsen, 2016b). The "unresolved mysteries about the growth process" listed by Galor (2005a, p. 220) have now been solved. They do not exist. They are phantom mysteries created by Galor through the manipulation of data.

Industrial Revolution had no impact on changing the trajectories of economic growth and of the growth of population. There was no population explosion. What is perceived as takeoffs or explosions are just the natural continuations of hyperbolic growth (Nielsen, 2014). There was also no "mind-boggling" and "perplexing phenomenon of the Great Divergence in income per capita across regions of the world in the past two centuries" (Galor, 2005a, pp. 177, 220).

Recently, economic growth and the growth of human population started to be diverted to slower trajectories but these new trajectories continue to increase close to the historical hyperbolic trajectories. Analysis of data shows that not only the Unified Growth Theory but also the Demographic Transition Theory, which is based on similar assumptions, is repeatedly contradicted by empirical evidence (Nielsen, 2016c).

All these features suggest new lines of investigation aimed at answering many important questions about economic growth and about the growth of human population. Why the economic growth and the growth of human population were hyperbolic. Why the hyperbolic growth was so remarkably stable over such a long time in the past. Why was it not affected by many random forces that were no doubt present? Why the economic growth and population growth trajectories were not affected by the Industrial Revolution. The only exception where there is a correlation between the Industrial Revolution and the economic growth and the growth of population is Africa, the poorest region. This boosting can be explained by the colonisation of Africa rather than by the beneficial effects of the Industrial Revolution. What models should be used to explain the historical hyperbolic economic growth and the growth of human population? What are the common features that link these two processes? Why was the economic growth and the growth of human population diverted relatively recently to new, non-hyperbolic trajectories? Are these new trajectories likely to change again into the apparently preferred hyperbolic growth? How to prevent such an undesirable event? What should be done to make the growth of population and economic growth sustainable?

Much work needs to be done but it would unwise and potentially dangerous to be guided by the Unified Growth Theory.

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3. Unified Growth Theory contradicted by the absence of impacts of the Industrial Revolution even in the United Kingdom

Introduction

It is generally believed that at a certain stage of human history, economic growth and the growth of population experienced a dramatic transition from the endless stagnation to a sustained growth (see for instance, Artzrouni & Komlos, 1985; Clark, 2003; Galor, 2005a, 2007; Galor & Weil, 2000; Hansen & Prescott, 2002; Klasen & Nestmann, 2006; Kögel & Prskawetz, 2001; Komlos, 2003; Manfredi & Fanti, 2003; Weiss, 2007). This dramatic event is described as the great escape from the Malthusian trap, as a take-off from stagnation to growth (Galor, 2005a, 2011) or as a transition to a "new stage" of "self-sustained growth" (Kögel & Prskawetz, 2001, p. 338). It was supposed to have been "a break from Malthusian equilibrium" (Clark, 2005, p. 1314), the escape characterised by "the unprecedented increase in population growth" (Galor, 2005b, p. 494), "the population sprint" (Thomlinson, 1965, p. 312) and the time when the "population growth accelerated" (Kögel & Prskawetz, 2001, p. 338). It was presumably a massive "simultaneous take-off in economic growth and population growth" (Kögel & Prskawetz, 2001, p. 338).

This supposed dramatic escape from the imagined Malthusian trap is claimed to have been strongly prompted and assisted by the Industrial Revolution. The rapid and far-reaching technological and sociological changes associated with this event are claimed to have been the driving force of the economic and demographic transition (Bar & Leukhina, 2005; Clark, 2005; Galor, 2005a; Galor & Mountford, 2006; Goodfriend & McDermott, 1995; Khan, 2008; Komlos, 1989, 2003; Lucas, 2002; Manfredi & Fanti 2003; Mataré, 2009; Šimurina & Tica, 2006; Tamura, 2002; Weiss, 2007). Clark claims that the Industrial Revolution "represented a break from the Malthusian equilibrium" (Clark, 2005, p. 1314). According to Weiss, Industrial Revolution "facilitated an endogenous take-off from the Malthusian trap" (Weiss, 2007, p. 327). Likewise, Komlos claimed that "The industrial revolution can therefore be conceptualized as a break out of the Malthusian demographic regime. It was a period of both economic and demographic expansion" (Komlos, 1989, p. 203). He wrote that "Industrial Revolution was also accompanied by an acceleration in population growth" (Komlos, 2003, p. 18). "The Industrial Revolution drove the demographic transition" (Khan, 2008, p. 9). It "brought in its wake an accelerated growth in the size of human populations" (Mataré, 2009, p. 381). According to Galor, "In the first phase of the Industrial Revolution, prior to the implementation of significant education reforms, physical capital accumulation was the prime engine of economic growth" (Galor, 2005a, p. 212).

All such claims, descriptions and explanations of the past growth are not based on a scientific analysis of relevant data but on a good dose of fantasy. They may sound plausible but they have to be accepted by faith. Science has no room for such dubious speculations. Inevitably, when faith is defended, contradicting data are either ignored or suitably manipulated to support the preconceived ideas (Ashraf, 2009; Galor, 2005a, 2005c, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008).

Remarkably, however, precisely the same data, when closely analysed, demonstrate that the preconceived ideas are incorrect (Nielsen, 2014, 2015a, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g). The established knowledge in demography and economic research, the knowledge revolving around the concept of Malthusian stagnation and around the concept of the escape from the supposed Malthusian trap is not based on the scientific process of investigation (Nielsen, 2016h).

In science, even one contradicting evidence in data is sufficient to question contradicted interpretations and then to try to revise them or even reject them, but in the case of the historical economic growth and of the growth of population we now have more than one contradicting evidence (e.g. Biraben, 1980; Clark, 1968; Cook, 1960; Durand, 1967, 1974, 1977; Gallant, 1990; Haub, 1995; Kapitza, 2006; Kremer, 1993; Livi-Bacci, 2007; Maddison, 2001, 2010; McEvedy & Jones, 1978; Nielsen, 2014, 2015a, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016i, 2016j; Podlazov, 2002; Shklovskii, 1962, 2002; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; von Foerster, Mora & Amiot, 1960; von Hoerner, 1975).

We have carried out extensive investigation of the leading postulates used to explain the historical growth of human population and the historical economic growth (Nielsen, 2014, 2015a, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016i, 2016j). In particular, we have demonstrated that within the range of analysable data, economic growth and the growth of population were hyperbolic. There was no Malthusian stagnation and there was never a takeoff from stagnation to growth, which could be described as the escape from the Malthusian trap because there was no trap in the economic growth and in the growth of population.

The range of analysable data describing economic growth, global, regional and national, extends down to AD 1 (Maddison, 2001, 2010) but for the world economic growth it was extended down to 1,000,000 BC (De Long, 1998). Maddison's data and the extended estimates show clearly that there was never stagnation in the economic growth.

For the growth of human population, regional and national estimates also extend only down to AD 1 (Maddison, 2001, 2010) but for the global growth, many estimates are available extending down to 10,000 BC (Biraben, 1980; Clark, 1968; Cook, 1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 2007; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994, United Nations, 1973, 1999, 2013). They were also extended down to 1,000,000 million years ago (Deevey, 1960; cited by Kapitza, 2006, Kremer, 1993 and Livi-Bacci, 2007). From the distance of one million years it does not really matter whether it is a million years ago or million years BC. The evidence again is clear and consistent: there was no Malthusian stagnation but a steadily-increasing growth, interrupted only twice in the past million years, or maybe three times if we count the minor disturbance between AD 1200 and 1400 (Nielsen, 2016j). Each time, population growth was diverted from one hyperbolic growth to another. Our analysis, which is in harmony with earlier research (Kapitza, 2006; Kremer, 1993; Podlazov, 2002; Shklovskii, 1962, 2002; von Foerster, Mora & Amiot, 1960; von Hoerner, 1975), shows that fundamental postulates accepted by the established knowledge in demography and in economic research are repeatedly and clearly contradicted by data.

We have also demonstrated that the Industrial Revolution, 1760-1840 (Floud & McCloskey, 1994) had no impact on shaping the trajectories of economic growth and of the growth of population. Now we are going to demonstrate that Industrial

Revolution had absolutely no impact on shaping the growth of population and the economic growth in the United Kingdom, the very centre of this revolution where its effects should be most convincingly demonstrated.

In our diagrams, population data will be expressed in billions while the data for the Gross Domestic Product (GDP) in billions of 1990 International Geary-Khamis dollars. The GDP per capita (GDP/cap) values will be expressed in 1990 International Geary-Khamis dollars.

Analysis of the growth of population in the UK

Hyperbolic growth can be uniquely identified by studying the reciprocal values of data (Nielsen, 2014) because hyperbolic growth is then represented by a decreasing straight line. For a sufficiently large range of data, if they follow a decreasing straight line, the growth is hyperbolic. In such displays, it is also easy to identify even small deviations from hyperbolic distributions because deviations from a straight line are easy to notice. In particular, any boosting in the economic growth or in the growth of population, such as the expected boosting caused by the Industrial Revolution should be readily identified.

For the reciprocal values, effects are reversed. A boosting of growth is indicted by a clear change of the trend in the *downward* direction while a diversion to a slower trajectory is indicated by an *upward* bending in the growth trajectory. Results of our analysis of population data (Maddison, 2010) in the United Kingdom are presented in Figures 1-3.

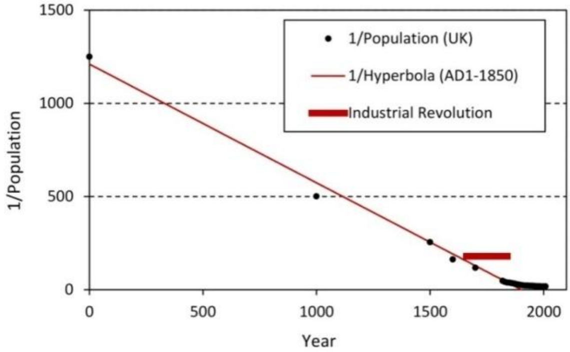

Figure 1. *Hyperbolic growth of population in the UK between AD 1 and 1850 as demonstrated by the decreasing straight line fitting the reciprocal values of the population data. Industrial Revolution did not boost the growth of the population in the UK. On the contrary, it coincided with the commencement of the gradually slowing down growth.*

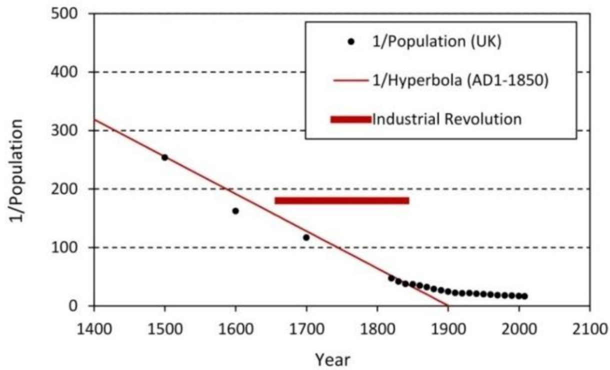

Figure 2. *The end part of the plot presented in Figure 1 showing that from around 1850, just at the end of the Industrial Revolution, the growth of the population in the UK started to be diverted to a slower trajectory. Industrial Revolution did not boost the growth of population in the UK.*

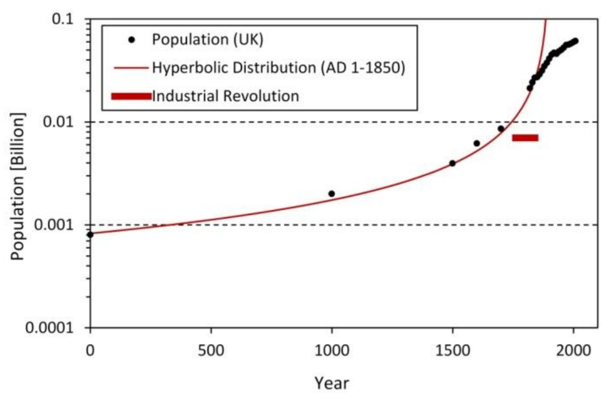

Figure 3. *Growth of population in the UK between AD 1 and 2008. Growth was hyperbolic between AD 1 and 1850. From around 1850, towards the end of the Industrial Revolution, the growth of population started to be diverted to a slower trajectory. Industrial Revolution had no impact on shaping the growth trajectory.*

In Figure 1 we show the reciprocal values of population data. Between AD 1 and 1850 they follow closely a decreasing straight line, showing that the growth of population was hyperbolic. Within the range of analysable data, which extends down to AD 1, the mythical epoch of Malthusian stagnation did not exist in the UK. The proof of the existence of Malthusian stagnation would have to be based on the demonstration of the existence of Malthusian oscillations. The data displayed in Figure 1 follow closely an undisturbed linear distribution representing an undisturbed and stable hyperbolic trajectory, indicating that even if random Malthusian forces were present, they had no effect on changing the growth trajectory. Any assumption of the presence of such forces is irrelevant.

It is also clear that the Industrial Revolution, 1760-1840 (Floud & McCloskey, 1994) had absolutely no impact on changing the growth trajectory. Data displayed in Figures 1-3 show clearly that there was no often-claimed boosting in the growth of population, no sprinting, explosion or any form of strong acceleration. On the contrary, from around 1850, shortly after the Industrial Revolution, the growth of population started to be diverted to a slower trajectory as indicated by the upper bending of the trajectory of reciprocal values shown clearly in Figure 2 and by a clear deviation from the hyperbolic trajectory shown in Figure 3.

These are remarkable results because the UK was in the centre of the Industrial Revolution. It is here that the effects of this revolution should be most strongly and most convincingly demonstrated but the data are in the direct contradiction of such expectations. It seems obvious that Industrial Revolution brought about many changes in the style of living and in social interactions, beneficial or detrimental, but all these changes had no effect on the growth of human population. It is as if this monumental event never happened.

Hyperbolic growth is described by a simple formula:

$$S(t) = \frac{1}{a - kt} \tag{1}$$

where $S(t)$ is the size of the growing entity (in our case either population or the GDP), while a and k are positive constants.

The increasing hyperbolic distribution, which could be also called the first-order hyperbolic distributions, is just the reciprocal of a decreasing straight line. That is why, a decreasing straight line of the reciprocal values identifies uniquely the first-order hyperbolic distribution.

Parameters of the hyperbolic distribution shown in Figure 3 are: $a = 1.210 \times 10^{3}$ and $k = 6.366 \times 10^{-1}$. Its singularity is at $t = 1901$. If continued along its historical trajectory, the growth of the population in the UK would have escaped to infinity in 1901. Fortunately, from around 1850 is started to be diverted to a slower trajectory bypassing the singularity by a safe margin of 51 years.

Analysis of the economic growth in the UK

Results of mathematical analysis of the historical GDP data for the UK are presented in Figures 4-6, while for the historical income per capita (GDP/cap) they are shown in Figure 7.

Reciprocal values displayed in Figure 4 show that the growth of the GDP was at first increasing along a fast, hyperbolic trajectory, as shown by a steep straight line fitting the reciprocal values of data. However, from around AD 1600, i.e. about 160 years *before* the commencement of the Industrial Revolution, the growth of the GDP was diverted to a *slower* hyperbolic trajectory as indicated by a less-steep straight line. This slower trajectory remained totally unaffected by the Industrial Revolution. This event did not even manage to revert the economic growth to the state experienced before AD 1600, when the hyperbolic trajectory was significantly faster. This slower hyperbolic growth continued until around AD 1850 when it started to be diverted to even slower trajectory indicated by an upward bending shown in Figure 5.

There was definitely no boosting in the economic growth caused by or associated with the Industrial Revolution. There was even no visible delay in the diversion to a slower trajectory. Industrial Revolution had no effect on the economic growth trajectory.

Again, these results are remarkable because the UK was right at the centre of the Industrial Revolution and it should have experienced its strong effect on the economic growth and on the growth of population. Technological and sociological changes brought about by the Industrial Revolution were present but, surprisingly perhaps, they did not accelerate the economic growth. It was as if economic growth were prompted by some other, much stronger force, which overruled any possible impacts of the Industrial Revolution. It would be interesting to identify this force but it is clear that the usual explanations based on the hypothetical forces of Malthusian stagnation and on the equally hypothetical forces of the Industrial Revolution, including the forces of technological development, are irrelevant for explaining the mechanism of the historical economic growth.

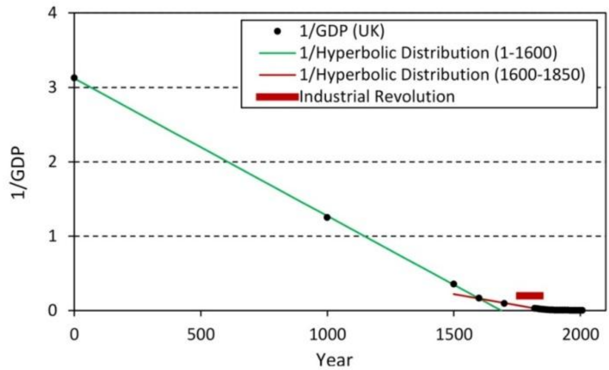

Figure 4. *Reciprocal values of the GDP data for the UK (Maddison, 2010) are compared with the decreasing linear distributions representing hyperbolic growth. The growth of the GDP was following a fast-increasing hyperbolic distribution (represented by a fast decreasing straight line) until AD 1600. From around that year and until around AD 1850, the economic growth was following a slower hyperbolic trajectory. Within the range of analysable data, i.e. from AD 1, the mythical epoch of Malthusian stagnation did not exist.*

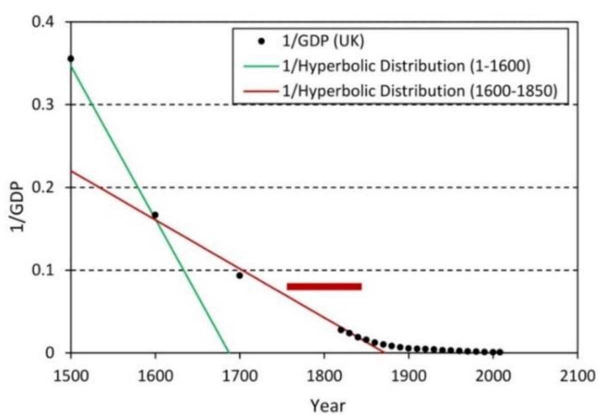

Figure 5. *The end part of the display shown in Figure 4. The slower hyperbolic growth, which commenced around AD 1600 (as indicated by the gently-decreasing straight line), continued undisturbed until AD 1850, i.e. throughout the entire time of the Industrial Revolution, which had absolutely no impact on the economic growth trajectory. There was no escape from the Malthusian trap because there was no trap. From around AD 1850, the growth of the GDP started to be diverted to a slower trajectory, as indicated by the upward bending of the reciprocal values trajectory.*

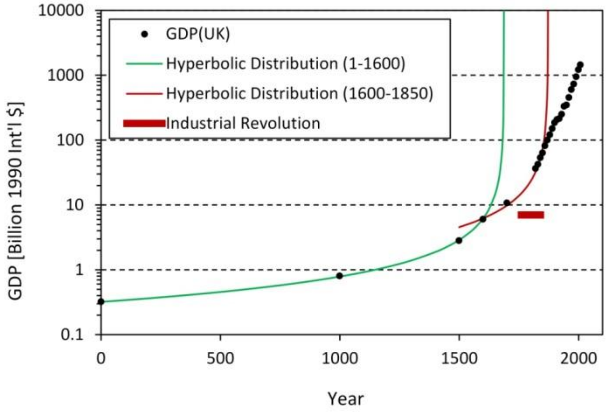

Figure 6. *Economic growth (as described by the GDP) in the UK. The growth was hyperbolic between AD 1 and 1600 and again (but a little slower) between AD 1600 and 1850. From around 1850, the growth started to be diverted to a slower but non-hyperbolic trajectory. Within the range of analysable data, i.e. from AD 1, the mythical epoch of stagnation did not exist. Economic growth was steadily increasing. Industrial Revolution did not boost the economic growth. There was no escape from the Malthusian trap because there was no trap.*

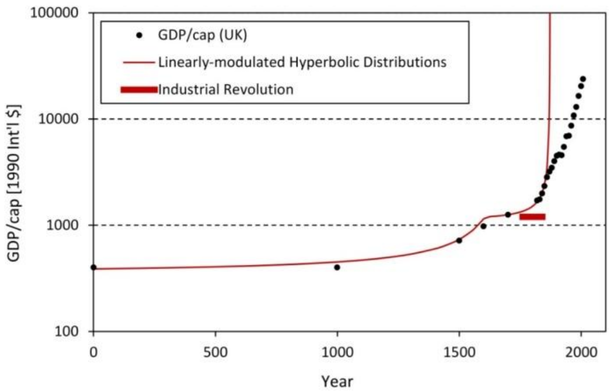

Figure 7. *Growth of income per capita (GDP/cap) in the UK between AD 1 and 2008. The GDP data follow closely the empirically-determined linearly-modulated hyperbolic distributions (defined in Nielsen, 2015a). Industrial Revolution did not change the growth trajectory. From around 1850, the growth of the GDP/cap started to be diverted to a slower trajectory.*

Hyperbolic fits to the GDP data are shown in Figure 6. The fast hyperbolic growth between AD 1 and 1600 is described by

$a = 3.120 \times 10^0$ and $k = 1.849 \times 10^{-3}$ Its singularity is at $t = 1687$. Contrary to the doctrine of Malthusian stagnation, economic growth was remarkably fast. If continued, it would escape to infinity about 73 years *before* the commencement of the Industrial Revolution. The slower hyperbolic growth of the GDP, which commenced in around AD 1600 is described by $a = 1.106 \times 10^0$ and $k = 5.909 \times 10^{-4}$. Its singularity is at $t = 1872$. This was also a steadily-increasing economic growth at the time when it was supposed to have been stagnant.

Population and economic growth data for the UK, and in particular the relatively fast economic growth before AD 1600, show how absurd is the concept of Malthusian stagnation. This concept is consistently contradicted by the analysis of other data describing economic growth and the growth of population (Kapitza, 2006; Kremer, 1993; Nielsen, 2014, 2015a, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016i, 2016j; Podlazov, 2002; Shklovskii, 1962, 2002; von Foerster, Mora & Amiot, 1960; von Hoerner, 1975).

The data for the UK show also how absurd is the doctrine of the boosting effects of the Industrial Revolution. The GDP and GDP/cap were already following fast-increasing trajectories *before* the Industrial Revolution. If continued, economic growth would escape to infinity in AD 1872. Any boosting by the Industrial Revolution would have been disastrous. Fortunately, natural processes did not comply with this ludicrous concept. Economic growth in the UK was not boosted by the Industrial Revolution but it was soon diverted into a slower pathway.

The same argument applies to the global and regional economic growth and to the global and regional growth of population. Propelled by the historical hyperbolic growth, they are now increasing too fast. Any boosting by the Industrial Revolution, any differential timing of the supposed takeoffs claimed by Galor (2005a, 2011), would be disastrous because it would propel economic growth and the growth of population along even faster trajectories and would render them unmanageable. Even now, we are approaching a serious global crisis but with the mechanism of growth approved by the established knowledge, this crisis would have occurred much earlier.

Nature or naturally occurring process take no notice of what we think is logical. Imagination is important in science but imagination has to be checked by meticulous analysis of data. We can propose convincing explanations but what we think as convincing is not necessarily what is reflected in the real world.

Scientific research has to be conducted scientifically; otherwise it is not scientific.

We can write as many fiction stories as we can possibly imagine them. They can be interesting and attractive but they have no place in science. Any theory that cannot be checked by data is regarded as unscientific and any theory that is contradicted by data has to be modified or even rejected and replaced by a new theory. Deliberately distorting the presentation of data (Ashraf, 2009; Galor, 2005a, 2005c, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008) to make them comply with preconceived ideas is not only unscientific but also self-defeating – we learn nothing from such mutilations of scientific evidence.

Doctrines of Malthusian stagnation and of the dramatic impacts of the Industrial Revolution on the growth of population and on the economic growth are repeatedly and consistently contradicted. These two doctrines and all the associated explanations and elaborate descriptions have no place in the economic and demographic research and the sooner they are abandoned the better. The continuing use of these doctrines to explain the historical economic growth and the historical growth of population is scientifically unjustified.

Defined by the parameter k, hyperbolic growth between AD 1 and 1600 was about three times faster than the hyperbolic growth between AD 1600 and 1850. The mythical epoch of stagnation did not exist. The transition around AD 1600 was not the usually-imagined transition from stagnation to growth but from growth to growth. It was not boosting but a transition from a fast to a slower hyperbolic growth. There is absolutely no expected correlation between the economic growth in the UK and the Industrial Revolution. No expected boosting and no transition from stagnation to growth because there was no stagnation. The wished-for takeoff is replaced by a transition to a *slower* trajectory. The established knowledge in the economic research is spectacularly contradicted by data, which were expected to give the most convincing support for the generally accepted doctrines.

The data refuse to comply with the desired and wished-for interpretations of the mechanism of economic growth. There was no wished-for escape from the Malthusian trap because there was no trap in the economic growth. There was also no trap in the growth of population in the UK. The only way to defend the established knowledge is to reject the data for the UK but then we would have to reject also other data and their analysed (Biraben, 1980; Clark, 1968; Cook, 1960; Durand, 1967, 1974, 1977;

Gallant, 1990; Haub, 1995; Kapitza, 2006; Kremer, 1993; Livi-Bacci, 2007; Maddison, 2001, 2010; McEvedy & Jones, 1978; Nielsen, 2014, 2015a, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016i, 2016j; Podlazov, 2002; Shklovskii, 1962, 2002; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; von Foerster, Mora & Amiot, 1960; von Hoerner, 1975).

Results of mathematical analysis presented Figure 7 show that the growth of income per capita in the UK can be described by two linearly-modulated hyperbolic distributions. The trajectory was calculated by dividing two hyperbolic distributions fitting the GDP data between AD 1 and 1850 (see Figures 4-6) by the hyperbolic distribution fitting the population data between AD 1 and 1850 (see Figures 1-3). For the discussion of the linearly-modulated hyperbolic distributions see Nielsen (2015a).

The growth of income per capita follows closely the empirically-determined growth trajectory. Industrial Revolution had no impact on changing the linearly-modulated hyperbolic growth. From around 1850, shortly after this industrial event, the growth of income per capita started to be diverted to a slower trajectory.

Summary and conclusions

The United Kingdom was in the centre of the Industrial Revolution. It is, therefore, the perfect place to test the currently accepted concept that the Industrial Revolution boosted economic growth and the growth of population. This concept is closely linked with the concept of Malthusian stagnation and the concept of the escape from the Malthusian trap. All these props are used to explain the mechanism of the economic growth and of the growth of human population. We have already demonstrated that all these accepted interpretations are contradicted by data (Nielsen, 2014, 2015a, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016i, 2016j). Now, we have focused our attention of the centre of the Industrial Revolution.

We have analysed the data (Maddison, 2010) describing the growth of population, the growth of the GDP and the growth of the GDP/cap in the UK. We have demonstrated that the historical growth of population and of the GDP were hyperbolic. Consequently, the historical growth of income per capita (GDP/cap) was linearly-modulated hyperbolic (Nielsen, 2015a).

We have demonstrated that over the entire range of the mathematically-analysable data, which in this case extends down to AD 1, the epoch of Malthusian stagnation did not exist. The growth of the population and the economic growth were increasing steadily without any signs of Malthusian stagnation.

We have demonstrated that the Industrial Revolution had absolutely no impact on shaping the growth of population and the economic growth in the UK, the very centre of this revolution where its effects should have been most clearly demonstrated. Thus, we have demonstrated yet again that the often-claimed effects of the Industrial Revolution on shaping the growth of population or on shaping the economic growth are contradicted by data.

The established knowledge in demography and in economic research is scientifically unacceptable (Nielsen, 2016h). It is contradicted by data and it flies in the face of everything we know about the current economic and demographic problems, which need to be urgently solved.

There was no transition from stagnation to a sustained growth regime (Galor, 2005a, 2011). The past growth was stable and secure as demonstrated by the largely undisturbed hyperbolic distributions but now, even though it became diverted from the fast increasing hyperbolic distributions to slower trajectories, it is still too fast and consequently insecure (Nielsen, 2015b).We might still have a sustained economic growth and sustained growth of population but it is generally acknowledged that in the long run our sustained growth is unsustainable because for the first time in human history we have already reached and crossed the ecological capacity of our planet (WWF, 2010).

The currently accepted paradigm based on the concept of Malthusian stagnation, on the concept of the escape from the Malthusian trap and on the concept of the boosting effects of innovations and technological development as represented by the Industrial Revolution, by the progress in medicine and by the dramatic changes in the style of living, is not only scientifically untenable but it is also potentially dangerous because it propagates the idea that after the endless epoch of stagnation we have now entered at last the sustained growth regime (Galor, 2005a, 2011). The real world in different. We have not escaped a Malthusian trap because there was no trap in the growth of population and in the economic growth. However, after the ages-long stable and secure growth, our current growth is no longer sustainable. For the first time in human history we have found ourselves in the trap of the fast-increasing economic growth and in the fast-increasing growth of population.

The erroneous traditional interpretations of economic growth and of the growth of human population are well illustrated in the

Unified Growth Theory (Galor, 2005a, 2011) based firmly on these incorrect concepts. In conformity with the traditional interpretations, Galor divided economic growth and the growth of population into three regimes: the Malthusian Epoch, the Post-Malthusian Regime and the Sustained Growth Regime. Economic growth and the growth of population is then explained using various complicated mechanisms, different for each of the imagined regimes. These erroneous concepts are supported by suitably distorted presentations of data (Ashraf, 2009; Galor, 2005a, 2005c, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008).

Data are never analysed. In their distorted presentations they appear to support the erroneous concepts based on impressions and on a good dose of fantasy. However, when analysed, precisely the same data show that the traditionally accepted doctrines have no support in science. Furthermore, they suggest that the mechanism of the historical economic growth and of the growth of population must be simple because they are described by the exceptionally simple mathematical distributions.

While the paradigm based on the concept of the endless epoch of stagnation followed by a sustained growth regime, creates a sense of security and prosperity, the data show that the opposite is true. It was the past growth that was safe and secure because it is described by the generally steadily increasing trajectories. However, now, for the first time in human history, our economic growth and the growth of human population is uncertain and insecure. We might reach a certain maximum in the growth of human population during the current century but we might not (Nielsen, 2006, 2015b, 2016h). The future is far from certain.

Interpretations of the mechanism of the historical economic growth and of the historical growth of population have to be based on data, and data are in the direct contradiction of the currently accepted paradigm (Nielsen, 2014, 2015a, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016i, 2016j). These interpretations have to be based on accepting hyperbolic growth. There is no choice: the traditional paradigm based on the concept of Malthusian stagnation followed by a distinctly new regime of sustained growth has to be replaced by the evidence presented by data that the past growth was hyperbolic but that, relatively recently, it was diverted to new trajectories.

Hyperbolic distributions may be confusing. They may create an illusion of stagnation followed by an explosion but this illusion is not a valid excuse for creating the whole system of scientifically unsupported doctrines and interpretations because the analysis of hyperbolic distributions is trivially simple (Nielsen, 2014). Anyone can do it and see that the currently accepted paradigm based on the assumption of the existence of Malthusian stagnation followed by the supposed escape from the Malthusian trap has no scientific support.

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4. Puzzling features of income per capita distributions explained

Introduction

Hyperbolic distributions appear to be creating significant problem with their interpretation. They are routinely seen as being made of two distinctly different components, slow and fast, jointed perhaps by a transition stage. However, these distributions are easy to understand if they are represented by their reciprocal values (Nielsen, 2014) because in this representation the confusing features disappear and hyperbolic distributions are represented by straight lines.

It is always convenient to reduce the analysis of data to a straight line, if possible, for two reasons: (1) straight lines are easy to understand and (2) any deviation from a straight line can be easily observed. For the exponential growth, the analysis can be reduced to a straight line by calculating the logarithm of data. For the hyperbolic growth, a straight line is produced by calculating the reciprocal values of data. However, for the income per capita, this simple method cannot be applied and we have to use a different approach. Furthermore, distributions describing income per capita are even more confusing than hyperbolic distributions because features, which were already difficult to understand for hyperbolic distributions, are even more confusing.

Incorrect interpretation of the historical GDP/cap data is a serious problem and the prominent example is the Unified Growth Theory (Galor, 2005a, 2011). Using the reciprocal values of the GDP data, it has been already demonstrated (Nielsen, 2014) that the fundamental postulates of this theory are contradicted by empirical evidence. We shall now demonstrate that the same conclusion can be reached by the analysis of the GDP/cap data coming from precisely the same source as used in developing this theory.

Unified Growth Theory tries to explain the apparent different stages of growth but we shall demonstrate that this explanation is grossly incorrect because the GDP/cap data follow a *single, monotonically increasing,* trajectory, which should be interpreted as a whole. We shall demonstrate that the three regimes of growth, postulated in the Unified Growth Theory and generally accepted in other related publications did not exist and that there was no generally claimed takeoff in the economic growth at any time.

Crude representation of data

The GDP/cap distributions are frequently displayed in a grossly simplified way by selecting just four strategically-located points (Ashref, 2009; Galor, 2005a, 2005b, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008) as shown in the top panel of Figure 1.

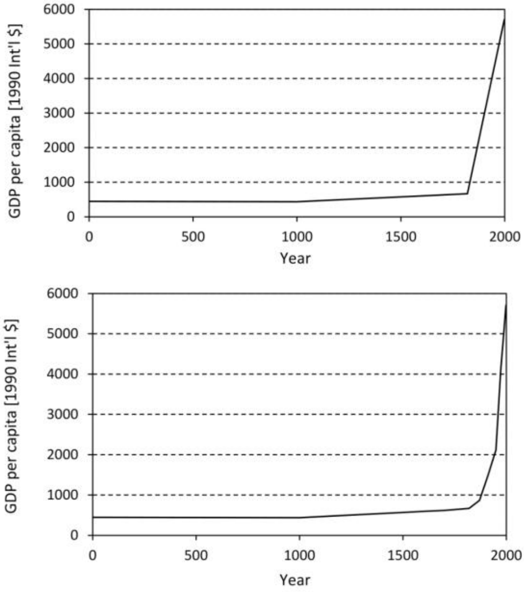

Figure 1. *Gross Domestic Product (GDP) per capita (Maddison, 2001) as frequently presented in numerous publications (Ashref, 2009; Galor, 2005a, 2005b, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). Strongly misleading impressions created by such presentations of data are the basis for promoting erroneous interpretations of the mechanism of economic growth and the prominent example is the Unified Growth Theory (Galor, 2005a, 2011)*

In this figure, we show an example for the world economic growth but similar plots are also used for regional data. Such displays are strongly suggestive and misleading, and they serve as a perfect prescription for drawing incorrect conclusions. This is a good example of the unscientific approach to research and it is hardly surprising that such handling of data leads to incorrect conclusions. Galor's Unified Growth Theory and all other associated publications are not based on science. They are unreliable and strongly questionable. Indeed, when closely analysed they are found to be repeatedly contradicted by data (Nielsen, 2014, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016h), even by the same data, which in their distorted way were used to support these numerous publications.

The GDP/cap distributions are already sufficiently confusing even if all data are plotted (see the lower panel in Figure 1). They do not have to be distorted to create even greater confusion. They have to be methodically and carefully analysed. Displays such as shown in Figure 1 are not helpful because they reinforce incorrect impressions and interpretations.

Impressions can be misleading and every effort should be taken to avoid being guided by their deception. Science is not based on impressions but on a rigorous analysis of data. Unified Growth Theory (Galor, 2005a, 2011) describes ideas based on impressions created by such displays as shown in Figure 1 or by quoting certain data without making any effort to analyse them scientifically. In this theory, many complicated but rather primitive mathematical formulations are presented, but incorrect concepts remain incorrect even if translated into mathematical formulae.

Explaining the GDP/cap ratio

The GDP/cap ratio combines two time-dependent distributions: (1) the time-dependent GDP growth and (2) the time-dependent population growth. In order to understand the GDP/cap distributions we have to understand their two components: the growth of the GDP and the growth of population.

Over 50 years ago, von Foerster, Mora and Amiot (1960) demonstrated that the world population was increasing hyperbolically during the AD era. Recent analysis shows that that the world population was increasing hyperbolically for thousands of years not only during the AD era but also during the BC era (Nielsen, 2016i). Hyperbolic growth of population applies not only to the global but also to regional populations (Nielsen, 2016d). Contrary to the expectation of Malthus (1798), when unchecked, population does not increase exponentially but hyperbolically.

Furthermore, the growth of population was hardly ever checked. Historical GDP values, global and regional, were also following hyperbolic distributions (Nielsen, 2014, 2016a, 2016h).

Even though hyperbolic distribution appears to be made of two different components, slow and fast, joined by a transition component, it has been shown (Nielsen, 2014) that such interpretation is based on strongly misleading impressions. Reciprocal values of a hyperbolic distribution describing growth follow a decreasing straight line and it is then obvious that it makes no sense to divide a straight line into arbitrarily selected sections and claim different mechanisms of growth for each section. It also makes no sense to look for a point marking a takeoff on such a monotonically decreasing straight line because a monotonically decreasing straight line remains a monotonically decreasing straight line and there is no justification in selecting a certain point on such a line and claim that there is a change of direction at this point because there is no change of direction.

In order to understand the GDP/cap distributions, the first and essential step in the past studies should have been to understand mathematical properties of their two components (GDP and population). Now we know that that they follow hyperbolic distributions. Consequently, in order to understand the historical GDP/cap data we have to understand the *mathematical process* of dividing two hyperbolic distributions.

We are going to demonstrate that the characteristic features of the GDP/cap distributions, which were used in the formulation of the Unified Growth Theory (Galor, 2005a, 2011), represent purely *mathematical properties* of dividing two hyperbolic distributions. They do not represent different socio-economic conditions describing different mechanisms of growth for different perceived sections of these distributions as claimed erroneously in the Unified Growth Theory.

Hyperbolic distribution describing *growth* is represented by a *reciprocal* of a linear function:

$$f(t) = (a - kt)^{-1}, \tag{1}$$

where $f(t)$ is the size of the growing entity, t is the time, and a and k are *positive* constants.

A reciprocal of hyperbolic distribution, $[f(t)]^{-1}$, is represented by a *decreasing* straight line:

$$[f(t)]^{-1} \equiv \frac{1}{f(t)} = a - kt \,. \qquad (2)$$

Hyperbolic *distributions* should not be confused with hyperbolic *functions* ($\sinh(t)$, $\cosh(t)$, etc). Furthermore, *reciprocal* distribution or functions, $[f(t)]^{-1}$, should not be confused with *inverse* functions, $f^{-1}(t)$. Mathematical symbol for the inverse function, $f^{-1}(t)$, is similar to the mathematical symbol for the reciprocal function, $[f(t)]^{-1}$, but the concepts are different.

In the inverse functions, the roles of variables are inversed. In the reciprocal functions, they remain the same. Thus, for instance, for the distribution given by the equation (1), the aim of using its inverse function would be to calculate how the *time* depends on the size of the growing entity. The inverse function of the eqn (1) is

$$f^{-1}(t) = \frac{a}{k} - \frac{1}{kt}, \qquad (3)$$

where t is now the size of the growing entity and $f^{-1}(t)$ is the time. For the reciprocal function given by the eqn (2), t is still the time as in the eqn (1). From the eqn (3) we can see that when the size of the growing entity, t, increases to infinity, the time, $f^{-1}(t)$, reaches its terminal value of a/k.

The characteristic feature of hyperbolic distributions is that they increase slowly over a long time and fast over a short time, escaping to infinity at a certain fixed time $t_s = a/k$, i.e. when the denominator in the eqn (1) approaches its zero value. However, as we have already pointed out and as discussed earlier (Nielsen, 2014), it is a mistake to interpret such distributions as being made of two distinctly different components joined by a transition component. It is one and continuous distribution, which has to be interpreted as a whole. If such a distribution represents a certain mechanism of growth, it is the same mechanism for the whole distribution.

Let us now take two, purely mathematical, hyperbolic distributions, $f(t)$ and $g(t)$, and let us divide them. Results are presented in Figure 2.

Parameters describing hyperbolic distributions displayed in Figure 2 are: $a = 4.5$ and $k = 2.2 \times 10^{-3}$ for $f(t)$ and $a = 7$ and $k = 3.35 \times 10^{-3}$ for $g(t)$. *These distributions are purely mathematical entities. They have nothing to do with the growth of the population or with the economic growth.* However, they satisfy a simple condition: the singularity of the $f(t)$ distribution occurs earlier than the singularity of the $g(t)$ distribution. For the curves displayed in Figure 2 singularities are at $t_s \approx 2045$ for $f(t)$ and $t_s \approx 2090$ for $g(t)$. The point of singularity for the $f(t)/g(t)$ ratio is, of course, at $t_s \approx 2045$.

When the distribution $f(t)$ is divided by $g(t)$ they produce a distribution, which resembles closely a typical GDP/cap distribution (see the lower panel of Figure 1). The characteristic features of this distribution are a long stage of nearly constant values of the $f(t)/g(t)$ ratio followed by a nearly vertical increase.

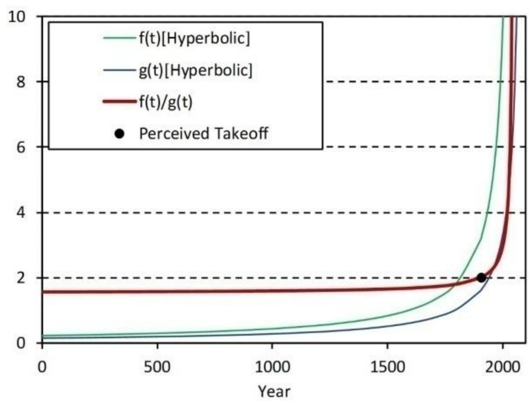

Figure 2. *Two, mathematically-defined, hyperbolic distributions,* $f(t)$ *and* $g(t)$*, and their ratio* $f(t)/g(t)$*. The time of the perceived, but non-existent, takeoff is indicated.*

It is important to notice that for the ratio of two hyperbolic distributions, the difference between slow and fast growth is much more clearly pronounced than for the corresponding hyperbolic distributions. The nearly horizontal part is flatter and the nearly vertical part is even more vertical. That is why, if the hyperbolic distributions are already so confusing, the distributions representing the ratio of two hyperbolic distributions are even more confusing and their interpretation is even more difficult. They have to be analysed with extra care and their analysis cannot be simplified by using their reciprocal values because the reciprocal of the ratio of two hyperbolic distributions is also a ratio of two hyperbolic distributions. Their analysis is significantly more difficult than the analysis of hyperbolic distributions. They represent a well-concealed trap suggesting strongly the existence of two or even three different components and even the most experienced researcher, who is not familiar with hyperbolic distributions or who is reluctant to accept them because of their singularity, can be easily misguided.

So we can see now that by dividing two, mathematically defined and monotonically increasing hyperbolic distributions, which have nothing to do with the economic growth, we have generated the fundamental features, which inspired the creation of the grossly incorrect Unified Growth Theory (Galor, 2005a, 2011) propagating such erroneous concepts as "the Malthusian Regime" represented by the flat "part," "Sustained-Growth Regime" represented by the steep "part," "the Post-Malthusian Regime," represented by the middle "part" and a "takeoff," represented by the apparent but non-existent fast transition from the flat to the steep growth. All these “parts” and the takeoffs do not exist because distributions representing the ratios of monotonically increasing hyperbolic distributions increase also monotonically. We could devote volumes on discussing the mechanism of growth of these imagined “parts” and trying to explain the triggering mechanism of the non-existent takeoffs but our discussions would have no scientific merit. Unified Growth Theory is made of such unscientific explanations but we can find them in numerous other publications, all creating the undesirable confusion and all of them diverting attentions from the correct interpretation of the mechanism of economic and population growth.

The puzzling and apparently peculiar features observed in the GDP/cap distributions can be reproduced using purely mathematical, monotonically-increasing, hyperbolic distributions. These features reflect purely mathematical properties of a *single* distribution representing the $f(t)/g(t)$ ratio. *They do not describe different stages of growth.* Furthermore, it is clear that these features cannot be attributed uniquely to the GDP/cap distributions. The division of two hyperbolic distributions may represent a certain mechanism of growth but it is still a *single* mechanism.

We have created an unusual and perhaps puzzling distribution but it would be incorrect to be so mesmerised by this simple mathematical operation as to propose different regimes of growth for different perceived parts of the $f(t)/g(t)$ ratio. We can see that the features observed for the GDP/cap distributions can be easily replicated by dividing two mathematically-defined hyperbolic distributions. It is, therefore, clear that hasty assumptions about different socio-economic conditions for the different perceived "parts" of the GDP/cap distributions can be questioned, which means that the whole Unified Growth Theory based on such assumptions can be not only questioned but indeed shown to be grossly incorrect and scientifically unacceptable (Nielsen, 2014, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016h). There is no point in presenting elaborate descriptions of different socio-economic conditions if these descriptions are contradicted by data. Even if the described socio-economic conditions did exist, they obviously had no impact on shaping economic growth trajectories, at least as expressed by the GDP or by the GDP/cap values. Such theories, as the Unified Growth Theory, could be regarded as interesting collections of stories but these stories do not assist in understanding the mechanism of economic growth.

The next step in explaining the GDP/cap distributions is now to explain why the division of two hyperbolic distributions generates such a puzzling trajectory, which appears to be made of two distinctly different components and why these apparently different components are so strongly pronounced.

Explaining the ratio of hyperbolic distributions

Using the eqns (1) and (2) we can see that the ratio of two hyperbolic distributions can be represented also in two other ways:

$$\frac{f(t)[Hyperbolic]}{g(t)[Hyperbolic]} = [g(t)]^{-1}[Linear] \cdot f(t)[Hyperbolic] = \frac{[g(t)]^{-1}[Linear]}{[f(t)]^{-1}[Linear]}. \quad (3)$$

These operations are represented graphically in Figure 3. We can see that all these mathematical operations create the same distribution representing the ratio $f(t)/g(t)$. It does not matter which pathway we take – results are the same.

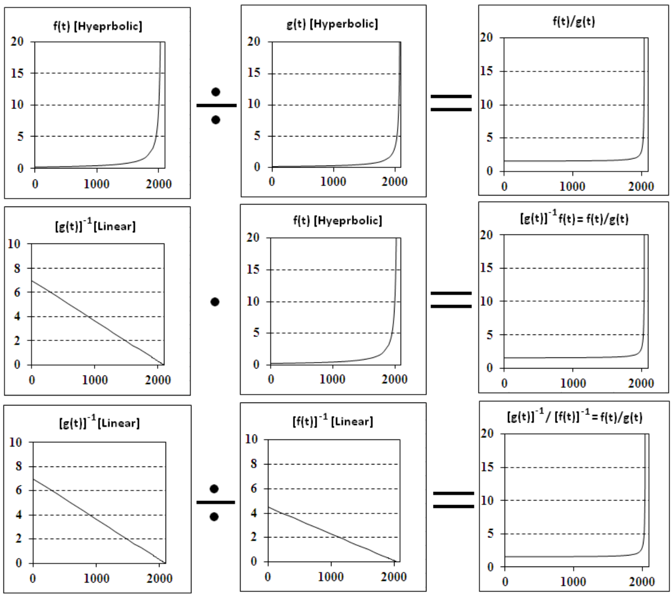

Figure 3. *Graphic representation of the eqns (3).*

Dividing two monotonically-increasing hyperbolic distributions is the same as multiplying hyperbolic distribution by a decreasing linear function and the same as dividing two decreasing linear functions. It is all just as simple as that. There are no hidden mysteries that need to be explained by some kind of complicated theories and mechanisms, but we still want to understand why these simple operations generate such a peculiar distribution, which appears to be made of two distinctly different components: horizontal and vertical.

The easiest way to understand the division of hyperbolic distributions is probably by looking at the middle section of Figure 3. The effect of the multiplication of hyperbolic distribution by the decreasing linear function is to lift up the left-hand part of the slowly increasing section of hyperbolic distribution and suppress the right-hand part. However, if $f(t)$ escapes to infinity earlier than $g(t)$, $f(t)$ will be escaping to infinity when $[g(t)]^{-1}$ is still positive. The values of $[g(t)]^{-1}$ will be small but the multiplication of the rapidly increasing values of $f(t)$ by small values of $[g(t)]^{-1}$ will have no effect on the escape to infinity. The product of such numbers will be also rapidly escaping to infinity. The combined effect of such a multiplication of a decreasing straight line by the increasing hyperbolic distribution is to flatten the slowly increasing section of the hyperbolic distribution without significantly changing the large values. The initial slow increase is made even slower and the *perceived* transition to the steep part is even more pronounced. However, *there is no mathematically-defined transition at any time* between these two perceived components.

The ratio of two hyperbolic distributions can be described simply as *the linearly-modulated hyperbolic distribution*. Thus, in our example the ratio of $f(t)/g(t)$ can be described as *the linearly-modulated hyperbolic* $f(t)$ *distribution*. The linear modulation is done by the linear function $[g(t)]^{-1}$ representing the reciprocal values of the hyperbolic $g(t)$ distribution.

Likewise, the distribution representing the historical GDP/cap growth can be described as *the linearly-modulated hyperbolic GDP distribution*. The linear modulation is done by the linear distribution representing the reciprocal values of the hyperbolic distribution describing the growth of human population.

The ratio of two hyperbolic distributions looks as if being made of two different components, slow and fast, but it is still the same, uninterrupted, monotonically increasing distribution. It is still a *single* mathematical distribution. It is the distribution, which is not made of two different sections. It is the distribution that it is *impossible* to divide into two distinctly different parts represented by two different functions. This distribution increases slowly over a long time and fast over a short time but the transition from the perceived slow to the perceived fast growth occurs *over the entire range of time*. It is *impossible* to determine the time of this perceived transition. It is impossible to determine the time of the perceived takeoff because *the takeoff does not exist* even if it appears to exist. The perceived takeoff is an illusion. There *is* a slow growth over a long time and a fast growth over a short time but there is no transition at any time between the slow and the fast growth. The slow and the fast growth are represented by the same, monotonically increasing distribution, which is not made of distinctly different components.

Even though the ratio of hyperbolic distributions, $f(t)/g(t)$, looks as if being made of two or three components (see Figures 2 and 3), even though the distribution represented by this ratio increases slowly over a long time and fast over a short time, even though it increases to infinity at a fixed time and even though it appears to be characterised by a takeoff at a certain time, it is still just a single, monotonically-increasing distribution, which is *impossible* to divide into different components. We have to accept it and learn to live with it.

Perhaps the easiest way to dispel the strong illusion of the distinctly different components of growth is to examinethe lowest part of Figure 3. It would be obviously unreasonable to claim that each of these straight lines is made of two or three distinctly different components, because these straight lines are obviously not made of different components. It would be unreasonable to claim different mechanisms of growth for various, arbitrarily-selected parts of these straight lines. At which point located on a straight line one mechanism of growth is supposed to end and a new mechanism to begin? It is *impossible* to claim two or three distinctly different sections on the monotonically decreasing straight lines. There is also obviously no feature on such straight lines that could be claimed as marking a takeoff.

We can also take a different approach and demonstrate again that the ratio $f(t)/g(t)$ represents a single, monotonically-increasing distribution and that there is no takeoff at any time. This different approach consists in calculating the gradient and the growth rate of the $f(t)/g(t)$ ratio. Results are presented in Figure 4 around the time of the perceived takeoff, i.e. when the $f(t)/g(t)$ reaches the value of 2 (see Figure 2). For better clarity, results are plotted as a function of the size of the $f(t)/g(t)$ ratio.

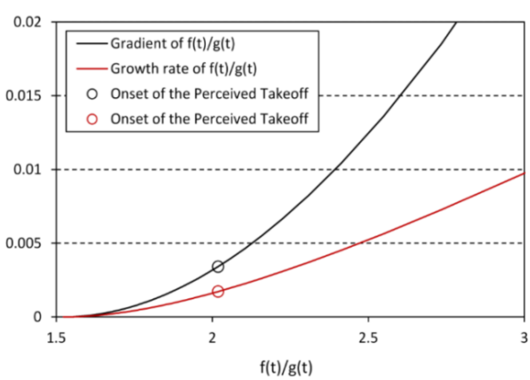

Figure 4. *The gradient and growth rate of the ratio of hyperbolic distributions* $f(t)/g(t)$*. The onset of the perceived takeoff shown in Figure 2 is indicated. This figure shows that the takeoff never happened and that the distribution representing the ratio* $f(t)/g(t)$ *is not made of different components. It is a single, monotonically-increasing distribution.*

These calculations show clearly that both the gradient and the growth rate of the hyperbolic ratio $f(t)/g(t)$ increase monotonically. The perceived takeoff never happened. What looks like a takeoff in Figure 2 is in fact just the continuation of the undisturbed and monotonically-increasing distribution representing the $f(t)/g(t)$ ratio. It is impossible to claim different components for any of the distributions displayed in Figure 4, representing the $f(t)/g(t)$ distribution, which in Figure 2 looks very deceptively as being made of two different components. It is impossible to claim a takeoff for any of these two distributions. The two components simply do not exist and the takeoff is just an illusion.

Analysis of the historical GDP/cap data

The GDP and population data (Maddison, 2001) [the same data as used but not analysed during the formulation of the Unified Growth Theory (Galor, 2005a, 2011)] together with their fitted hyperbolic distributions are shown in Figure 5. Indicated in the figure is the time of the Industrial Revolution 1760-1840 (Floud & McCloskey, 1994), which is generally claimed as the time of the supposed takeoff in the economic growth (Galor, 2005a, 2008a, 2011, 2012). Parameters fitting the GDP data are: $a = 1.716 \times 10^{-2}$ and $k = 8.671 \times 10^{-6}$ while parameters fitting the population data are $a = 8.724$ and $k = 4.267 \times 10^{-3}$.

Points of singularity are: $t_s \approx 1979$ for the world GDP and $t_s \approx 2045$ for the population data. The point of singularity for the world GDP is before the point of singularity for the growth of the world population. Consequently, the GDP/cap ratio should display the same features as shown in Figure 2 for the $f(t)/g(t)$ ratio and indeed, it does.

In Figure 6 we present the data for the GDP/cap and the corresponding fit to the data calculated by dividing the corresponding hyperbolic distributions shown in Figure 5. The calculated curve and the data shown in Figure 6 follow a similar distribution as displayed in Figure 2. The characteristic features of the nearly horizontal growth over a long time and the nearly vertical growth over a short time of the GDP/cap distribution are nothing more than the mathematical property of dividing two hyperbolic distributions.

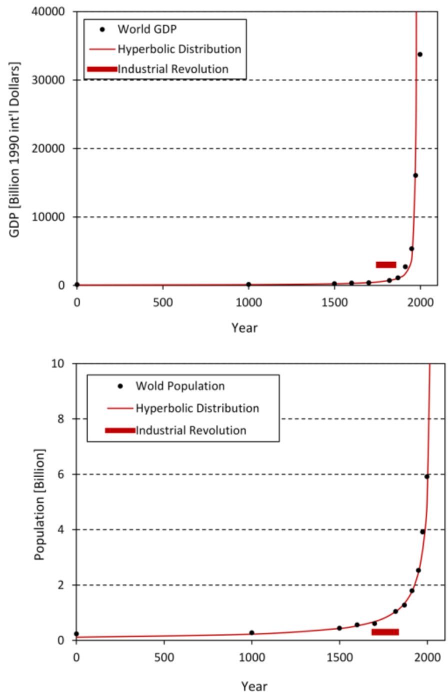

Figure 5. *Hyperbolic distributions are compared with the world GDP and population data (Maddison, 2001). The GDP is expressed in billions of 1990 International Geary-Khamis dollars and the population in billions.*

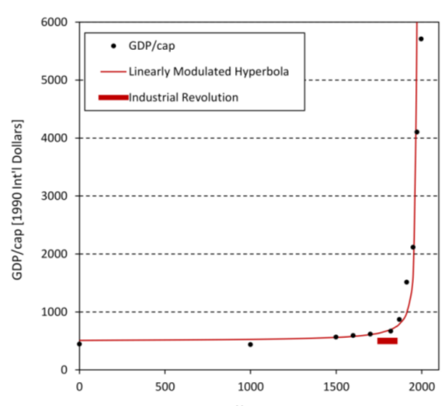

Figure 6. *Calculated linearly-modulated hyperbolic GDP distribution, representing the GDP/cap ratio, is compared with the world GDP/cap data (Maddison, 2001). The GDP/cap is expressed in the 1990 International Geary-Khamis dollars.*

If the point of singularity for the GDP trajectory was located higher than the point of singularity for the population trajectory, the growth of the GDP/cap would also have remained nearly constant over a long time but it would eventually decrease to zero at the time of the singularity for the growth of population. Income per capita would not have been increasing with the size of the population. On the contrary, it would have been decreasing. For hyperbolic distributions, the growth of income per capita depends on the relative positions of singularities of the two components.

According to Galor (2008a, 2012a), the so-called Malthusian Regime, represented presumably by the nearly constant income per capita, commenced around 100,000 BC. There is, of course, no justification for this date because the Malthusian Regime did not exist (Nielsen, 2014, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016h; von Foerster, Mora & Amiot, 1960). However, if we wanted to claim a certain date for this mythical regime, one would imagine that the usually claimed date of 200,000 BC for the onset of the existence of *Homo Sapiens* would have been more suitable.

He also claims that Malthusian Regime was terminated in AD 1750 for developed countries and in 1900 for less-developed countries. The Post-Malthusian Regime was supposed to have existed between 1750 and 1870 for developed countries and from 1900 for less-developed countries. The Sustained-Growth Regime was supposed to have commenced in 1870 and is supposed to continue until the present time. It is impossible to determine such specific landmarks for the monotonically increasing distributions. These imagined dates are contradicted by data. There were no takeoffs in the growth of the GDP, and the historical GDP trajectory cannot be divided into two or three different regimes (Nielsen, 2014, 2016b, 2016c). We also know that the growth of human population was hyperbolic and that it was never characterised by a sudden takeoff (Nielsen, 2016d, 2016i). Consequently, even though the GDP/cap data might be suggesting the existence of different stages of growth governed by different mechanisms of growth, their scientific analysis clearly demonstrates that different regimes of growth did not exist. Each historical GDP/cap distribution, global or regional, has to be interpreted as a whole and the same mechanism has to be applied to the slow and fast growth. Under these conditions, the interpretation of the mechanism of growth appears to be complicated because we have to use the same mechanism to explain the slow and fast growth. However, the explanation turns out to be exceptionally simple (Nielsen, 2016j).

By following our earlier approach, which we used for the division of arbitrary hyperbolic distributions, we can demonstrate that there was no takeoff in the GDP/cap distribution and that the three regimes of growth did not exist. We shall do this by calculating the gradient and the growth rate for the calculated GDP/cap trajectory. These calculations are presented in Figures 7 and 8.

A takeoff in the GDP/cap trajectory would be marked by a clear change in the gradient and in the growth rate around the time of the Industrial Revolution when a transition to a new economic growth regime was supposed to have happened (Galor, 2005a, 2008a, 2011, 2012a). The shape of the trajectories describing the gradient and growth rate would have to be distinctly different before and after the Industrial Revolution. There should be a certain clear discontinuity.

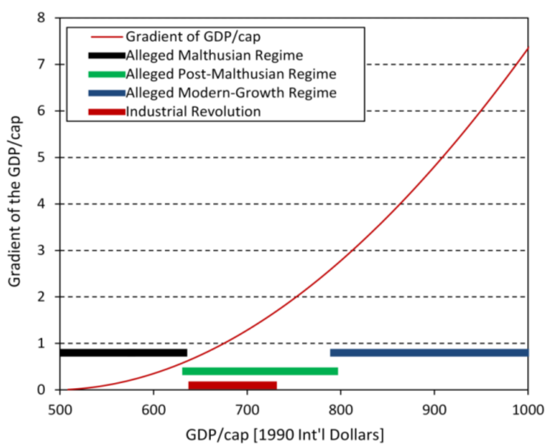

Figure 7. *Gradient of the world GDP/cap calculated using the fitted, linearly-modulated hyperbolic distribution shown in Figure 6. The GDP/cap is expressed in the 1990 International Geary-Khamis dollars. There was no takeoff at any time and the three regimes of growth postulated by Galor (2005a, 2011) did not exist.*

The gradient and the growth rate of the fitted curve increase monotonically confirming that the fitted, linearly-modulated hyperbolic distribution increases also monotonically. The calculated curve gives excellent fit to the GDP/cap data and consequently the gradient and the growth rate of the fitted curve represent also the gradient and the growth rate of the data.

Figures 7 and 8 clearly demonstrate that there is no reason for terminating the supposed Malthusian Regime around AD 1750 and for starting a new regime because there was no unusual change in the gradient and in the growth rate of the GDP/cap around that time, but there was also no scientifically-justified reason for assuming the existence of the Malthusian Regime. There is no reason for terminating the equally imaginary Post-Malthusian Regime around 1870 and starting the supposed Sustained-Growth Regime. There is no reason for slicing the monotonically-increasing distributions into three arbitrarily-selected sections. There is no reason for proposing three regimes of growth governed by distinctly different mechanism. There is no reason for claiming a takeoff at any time. There has been no scientifically justified reason for creating the Unified Growth Theory and three is no scientifically justified reason for adopting such concepts in the interpretations of economic growth and of the growth of population.

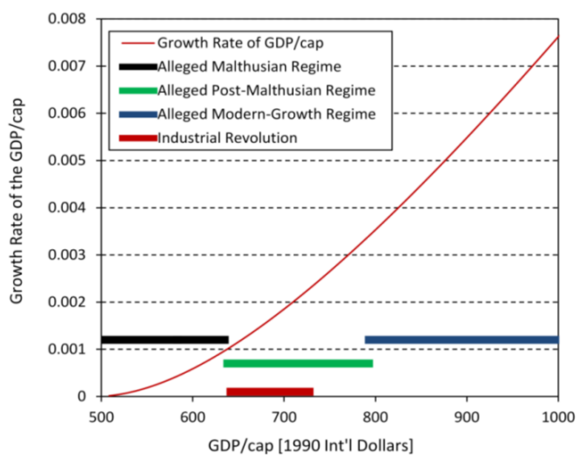

Figure 8. *Growth rate of the GDP/cap calculated using the fitted, linearly-modulated hyperbolic distribution shown in Figure 6. The GDP/cap is expressed in the 1990 International Geary-Khamis dollars. There was no takeoff at any time and the three regimes of growth postulated by Galor (2005a, 2011) did not exist.*

These calculations, supported by data, clearly demonstrate that the Industrial Revolution had no impact on the economic growth trajectory. Impacts were of different kind but the data show that the Industrial Revolution did not boost the global economic growth. It did not even boost the economic growth in Western Europe (Nielsen, 2014), or in any other region (Nielsen, 2016a, 2016g) or even in the United Kingdom (Nielsen, 2016h), the very centre of this revolution, where its effects on the growth trajectory should have been most clearly pronounced. There was no impact whatever on the growth trajectories. Economic growth must have been prompted and controlled by some other force, which was much stronger than any other forces, including the force of the Industrial Revolution and this force is discussed in a separate publication (Nielsen, 2016j). Furthermore, Galor's three regimes of growth did not exist.

Fundamental postulates of the Unified Growth Theory (Galor, 2005a, 2011) are contradicted by the analysis of data, the same data as used but not analysed during the formulation of his theory. Unified Growth Theory describes and explains phenomena that did not exist and consequently it does not explain the historical economic growth. It is an incorrect and misleading theory.

The discussion of socio-economic conditions presented by Galor might be interesting for some other reason but there is a clear evidence in the GDP and GDP/cap data that his discussion has no relevance to explaining the mechanism of economic growth. His discussed associations and correlations are not just questionable but plainly incorrect because they are contradicted by data he used but never analysed.

Economic growth was indeed slow over a long time and fast over a short time but it is incorrect to divide this monotonically increasing distribution into three regimes and claim distinctly different mechanisms for the arbitrarily selected sections. It is also incorrect to claim that there was a takeoff at a certain time. The data and their analysis give no scientific basis for such claims.

Historical economic growth has to be explained using a *single mechanism*. Such a mechanism should describe the slow and fast growth including the apparent transition. All these "parts" should be treated as one. Only then we could claim that we have explained the mechanism of the historical economic growth.

Dividing the past growth into three different regimes and claiming three different mechanisms is unsupported by data and it does not explain the mechanism of the historical economic growth. A truly unified growth theory will have to be based on a *single mechanism*. Such an explanation is proposed in a separate publication (Nielsen, 2016j).

Summary and conclusions

The aim of our discussion was to explain the puzzling features of the GDP/cap distributions. They show a slow growth over a long time, followed by a rapid increase. These features create a significant problem with their interpretations, and the outstanding example of the created confusion is the Unified Growth Theory (Galor, 2005a, 2011). Our discussion was based on precisely the same data which were used, but not analysed, during the formulation of this theory. It is both surprising and disappointing that while using excellent sets of data published by the world-renown economist (Maddison, 2001), Galor made no attempt to adopt scientific approach to developing his theory.

Historical economic and population growth, global and regional, show a clear preference for increasing along hyperbolic trajectories (Nielsen, 2014, 2016a, 2016d, 2016g, 2016h; 2016i). Hyperbolic growth contains singularity, when a growing entity escapes to infinity at a fixed time. We might think that such a growth is impossible but we have to accept the evidence in data. The past growth of the GDP and of population were hyperbolic. There is absolutely no problem with accepting hyperbolic growth for two reasons: (1) hyperbolic growth is obviously possible because it is demonstrated convincingly by data and (2) growth trajectories can change and there is nothing strange or unusual about it. Indeed, recently, hyperbolic growth was diverted to slower trajectories (Nielsen, 2016a, 2016d, 2016i).

It is remarkable, that this apparently impossible (because of its singularity) hyperbolic growth was possible for the most part of the past 12,000 years (Nielsen, 2016i). Every time it was interrupted, and it happened only twice in the past, it was converted again to a hyperbolic growth. Now, it is interrupted again but the future trajectory is yet unknown.

We have discussed mathematical properties of the historical GDP/cap distributions. We have explained how they should be analysed and interpreted.

If both components of the GDP/cap indicator increase hyperbolically, then the GDP/cap distributions represent a ratio of hyperbolic trajectories. We have a consistent evidence in data that the economic growth and the growth of population were hyperbolic (Nielsen, 2016a, 2016d, 2016i; von Foerster, Mora & Amiot, 1960). The characteristic features created by the division of hyperbolic distributions may be confusing but they can be easily explained. Data have to be analysed. Presenting them in a grossly distorted way is self-defeating and it leads to incorrect conclusions (Ashref, 2009; Galor, 2005a, 2005b, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008, Snowdon & Galor, 2008).

We have explained how to understand the confusing features of the historical GDP/cap distributions. They can be interpreted simply as the linearly-modulated hyperbolic GDP distributions.

Linear modulation is by the reciprocal values of population data. We have discussed how these distributions can be analysed, how their features can be explored and explained.

As an illustration of our discussion, we have investigated the data (Maddison, 2001) used in developing the Unified Growth Theory (Galor, 2005a, 2011). In his theory, Galor discusses various socio-economic concepts of growth but his theory does not explain the mechanism of economic growth because it is based firmly on the misinterpretation of the purely mathematical features of hyperbolic distributions. His discussion of socio-economic issues might be interesting, for various reasons, but it has no relevance to explaining the mechanism of the economic growth because changes in socio-economic conditions had no effect on the economic growth trajectory as manifested by the available data (Maddison, 2001), the same data, which were used, but not analysed, during the formulation of the Unified Growth Theory.

Galor's speculations about socio-economic processes are strongly guided by phantom features created by the deliberately distorted presentations of data (Ashref, 2009; Galor, 2005a, 2005b, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). Different stages of growth claimed in this theory did not exist. Their claimed presence is contradicted by the same data, which were used during the development of the Unified Growth Theory and in all other related publications, as listed above.

In general, the GDP and population, global, regional and even in individual countries, were increasing monotonically and consequently the GDP/cap ratios are also represented by monotonically increasing distributions governed by a single mechanism of growth.

Unified Growth Theory does not explain the historical economic growth because it is critically and inflexibly based on the deliberately constructed phantom features, which are contradicted by data. In particular, the three regimes of growth claimed by this theory did not exist and there were no takeoffs in the economic growth or in the growth of population. This theory describes a phantom world but presents it as real. Stories and explanations presented in the Unified Growth Theory might sound plausible but they are contradicted by data.

Historical GDP/cap distributions might look puzzling and complicated but they are in fact simple distributions. Their puzzling features are nothing more than just the mathematical features created by dividing two hyperbolic distributions. Their mechanism might also look complicated but hyperbolic distributions are described by exceptionally simple mathematical formula and the mechanism of these distributions, as representing the historical economic growth and the historical growth of population, is also simple (Nielsen, 2016j).

Historical hyperbolic economic growth can be explained as having been propelled by the simplest possible market force where the growth of the GDP (or the on average growth of the common wealth) is prompted by the force directly proportional to the already existing size of the GDP. On average, wealth was generating wealth directly proportionally to the existing wealth. Historical hyperbolic growth of the population can be explained as having been propelled by the simplest force of procreation (the combination of the natural sex drive combined with the natural process of aging and dying), which on average was constant per person. Historical growth of income per capita, expressed as the GDP/cap, can be explained as having been prompted by the combination of these two forces, and the puzzling features of the GDP/cap distributions turn out to be nothing more than the mathematical properties of dividing two hyperbolic distributions. If these simplest forces of growth are combined with some other strong forces, as it is now, the economic growth and the growth of population are no longer hyperbolic.

The current GDP/cap values are still increasing but the shapes of their distributions cannot be explained by the mathematical properties of diving two hyperbolic distributions because we are no longer dealing with hyperbolic distributions. However, in principle, their shapes could be reproduced by dividing mathematical distributions describing the current growth of the GDP and population. However, the underlying mechanism of any of them is now no longer simple and the mechanism of the *current* growth of income per capita is also no longer simple.

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5. Demographic Transition Theory contradicted by data

Introduction

Historical economic growth can be studied using the Gross Domestic Product (GDP). However, to understand the time dependence of income per capita, expressed as GDP/cap, it is necessary to understand not only the economic growth, expressed in terms of the GDP, but also the growth of human population.

The latest and the most elaborate theory describing economic growth is the Unified Growth Theory (Galor, 2005; 2011). The theory, or model, describing the growth of human population is the Demographic Transition Theory (see for instance Caldwell, 1976; 2006; Casterline, 2003; Coale, 1973; Haupt & Kane, 2005; Kirk, 1996; Landry, 1934; Lee, 2003; Lehr, 2009; McFalls, 2007; Notestein, 1945; Olshansky & Ault, 1986; Olshansky, Carnes, Rogers, & Smith, 1997, 1998; Omran, 1971; 1983; 1998; 2005; Rogers & Hackenberg, 1987; Singha & Zacharia, 1984; Thompson, 1929; van de Kaa, 2008; Warf, 2010). Both of these theories use similar approach and similar language. Both of them divide the economic growth or the growth of human population into distinctly different stages governed by distinctly different mechanisms. In particular, both of them claim an ages-long epoch of Malthusian stagnation followed by a sudden transition to a distinctly different stage, the transition described as a sudden takeoff, spurt, sprint or explosion.

A study published over 50 years ago (von Foerster, Mora & Amiot, 1960) demonstrated that the growth of the world population was hyperbolic during the AD era, showing implicitly that the epoch of stagnation did not exist and that there was no sudden transition to a new type of growth. This study has shown that the growth of human population during the AD era was following a monotonically increasing trajectory. As explained elsewhere (Nielsen, 2014), such a growth cannot be divided into distinctly different sections governed by distinctly different mechanisms of growth. A single mechanism has to be applied to the whole distribution. For reasons, which are hard to explain, this crucial publication (von Foerster, Mora & Amiot, 1960) appears to have been ignored in the demographic research.

More recently (Nielsen, 2016), it has been demonstrated that the growth of the world population was hyperbolic not only during the AD era, as pointed out by von Foerster, Mora and Amiot (1960) but also during the BC era. Furthermore, it has been demonstrated that there was no stagnation and consequently no transition from stagnation to a distinctly different and faster growth as claimed by the Demographic Growth Theory. This study identified only two transitions in the past but they were transitions of entirely different kind than claimed by the Demographic Transition Theory. They were transitions from hyperbolic growth to hyperbolic growth. The first transition was from a fast hyperbolic growth to a significantly slower hyperbolic growth and the second transition from a slow hyperbolic growth to a slightly faster hyperbolic growth. Thus, these two studies (Nielsen, 2016; von Foerster, Mora & Amiot, 1960) demonstrate that the Demographic Transition Theory is incorrect. Now we shall discuss additional evidence and we shall show that the Demographic Transition Theory is contradicted not only by the aggregate data describing the growth of the population but also by data describing birth and death rates.

Demographic Transition Theory has been described a ghost story (Abernethy, 1995). It should have been discarded long time ago but it is still in circulation and many a demographer would passionately defend its concepts. Abernethy wonders why this dead theory is still being resurrected and her plausible explanation is that it is because of the respect to elders. However, would elders feel happy to be so protected?

Science is full of discarded theories and explanations. This is how science works. New ideas are tried and if they do not work they are replaced by better ideas or simply abandoned. To cling to incorrect ideas just because we cannot think about something better to replace them is scientifically unjustified.

Friedman, Managing Editor of the *Population and Development Review*, claims that the Demographic Transition Theory with its "formulaic presentation of the four states" "is largely a straw man" (Friedman, 2015). This classical version of the Demographic

Transition Theory is now known as the first demographic transition to which a second demographic transition has been added (Lesthaeghe, 2010; 2014; Lesthaeghe & van de Kaa, 1986; van de Kaa, 2001; 2002). The classical four stages of growth are still there even though they have no convincing support in data."It is fair to say, that nearly all statements of a general kind about the classical - for me now the first - demographic transition, can be easily contradicted" (van de Kaa, 2002, p. 9). The classical Demographic Transition Theory appears to have been not only acknowledged but also reinforced by adding the international migration component. Kirk observed that "Demography is a science short on theory, but rich in quantification" (Kirk, 1996, p. 361) but it would be perhaps better to have science without a theory than "science" with a theory contradicted by data.

There is no science without data. In science, even the best constructed theory can be undermined and even abolished by just one contradicting evidence. It would be better to accept that it is perhaps impossible to have a general theory in the demographic research and that each case should be explained individually.

The curious feature of the Demographic Transition Theory is that *there is not a single convincing confirmation of this theory in data.* Try as we may, we shall never find data showing convincingly the four stages of growth. It is for this reason that Montgomery had to stitch the data for Sweden and Mauritius to *illustrate* this theory (Montgomery, n.d.). "I used Mauritius and added Sweden to the end of it. I smoothed the stage 1 of Mauritius a bit. It is composite more than purely conceptual" (Montgomery, 2012). It should be emphasised that his aim was not to prove this theory but only to illustrate it.

Data for Sweden are repeatedly used in support of the Demographic Transition Theory but we shall show that these data serve as an excellent illustration that the Demographic Transition Theory is contradicted by empirical evidence. Data for Mauritius are sometimes used but we shall show that they also do not support this theory. The best and the most extensive data are for England (Wrigley & Schofield, 1981). We shall demonstrate that the Demographic Transition Theory is also contradicted by these data.

It is taken for granted that the first stage, which is believed to have lasted for thousands of years, was characterised by strong fluctuations in birth and death rates but we have absolutely no data to prove it. We do not have data for death and birth rates extending over thousands of years, so in this sense at least this part of the theory is unscientific. We have no choice but to accept it by faith.

However, much more has to be accepted by faith. No-one has ever proven the existence of the first stage of growth (the epoch of stagnation) proposed by the Demographic Transition Theory. In fact, this concept is contradicted by data (Nielsen, 2013, 2016; von Foerster, Mora &Amiot, 1960). There was no stagnation in the growth of human population but for doctrines accepted by faith, contradictions in data are routinely and promptly ignored. The only way to accept this stage of growth is by faith and by ignoring population data (Maddison, 2010; Manning, 2008; US Bureau of Census, 2016) and their contradicting evidence, but then it is no longer science. Countless descriptions of this mythical epoch and of the mechanism of growth during that long time have to be accepted by faith.

No-one has ever proven that there was a transition from the first to the second stage. No-one has ever proven that there was population explosion at a certain time. Rapid growth of the population, interpreted as population explosion, is real but it is just the natural continuation of hyperbolic growth (Nielsen, 2014; 2016), the type of growth, which was identified over 50 years ago (von Foerster, Mora & Amiot, 1960) but which was also conveniently ignored. The transition from the supposed first to the second stage has to be accepted by faith and by ignoring not only the evidence published over 50 years ago but also the extensive population data (Maddison, 2010; Manning, 2008; US Bureau of Census, 2015).

No-one has ever proven that the mechanisms of growth during the supposed first and second stages were different. No-one has ever proven that the Industrial Revolution boosted the growth of human population. All these concepts and more have to be accepted by faith supported perhaps occasionally by the misinterpretation of selected data.

It is believed that strong fluctuations in birth and death rates are reflected in fluctuations in the size of the population. These assumed fluctuations, described often as Malthusian oscillations, have been extensively discussed in peer-reviewed literature but no-one cared to check whether fluctuations in birth and death rates have any influence on the growth of human population. We shall demonstrate that these *fluctuations have absolutely no impact on the growth of human population.*

It is believed that the growth of the population was stagnant for thousands of years and that it was characterised by random variations. According to this belief, there were periods of time when the population *did not grow at all* and that any gains in the growth of human population made over decades were *wiped out in* *one or two years* (van de Kaa, 2008). Such confident declarations are inaccurate and misleading. They might apply to some local populations, sometimes, but they certainly do not apply to the growth of the world population (Nielsen, 2013; 2016). This claim is also not supported by the regional population data (Maddison, 2010).

Normally, in any scientific investigation, empirical evidence such as published over 50 years ago (von Foerster, Mora &Amiot, 1960) would have been further investigated. Why was it ignored in the demographic research? This early observation is now convincingly confirmed (Nielsen, 2016) by new data (Maddison, 2010; Manning, 2008; US Bureau of Census, 2015). The growth of the population in the past was hyperbolic. It was slow but it was not stagnant or random. *The first stage proposed by the Demographic Transition Theory did not exist and there was no transition from stagnation to growth.*

Demographic Transition Theory

We have already mentioned certain features of the Demographic Transition Theory but in order to understand the discussed examples for Sweden, Mauritius and England we shall now present its brief outline. Its general concepts are illustrated in Figure 1.

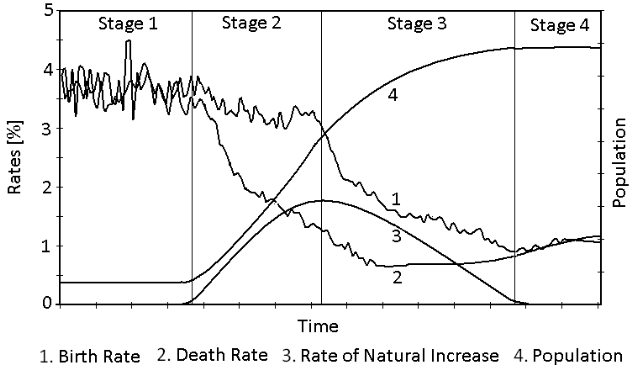

Figure 1. *Fundamental concepts of the Demographic Transition Theory based on the illustrations presented by Montgomery (n.d.) and by van de Kaa (2001; 2002).*

Demographic Transition Theory describes changes in birth and death rates, in the size of the population and in the rate of natural increase. According to this theory, changes in socio-economic conditions lead to transitions in death and birth rates, which in turn are reflected in the growth of human population (see for instance Caldwell, 1976; 2006; Casterline, 2003; Coale, 1973; Haupt & Kane, 2005; Kirk, 1996; Landry, 1934; Lee, 2003; Lehr, 2009; McFalls, 2007; Notestein, 1945; Olshansky & Ault, 1986; Olshansky, Carnes, Rogers, & Smith, 1997; 1998; Omran, 1971; 1983; 1998; 2005; Rogers & Hackenberg, 1997; Singha & Zacharia, 1984; Thompson, 1929; van de Kaa, 2008; Warf, 2010). These transitions are supposed to have been taking place in four fundamental stages, to which other stages could be added.

Stage 1 is supposed to have been the pre-industrial stage of stagnation; Stage 2 is supposed to represent the post-industrial stage of explosion; Stage 3 is the stage of the slowing-down growth; and Stage 4 is the stage of a stable size of the population. The number of stages can be extended to five (Haupt & Kane, 2005; Olshansky, Carnes, Rogers, & Smith, 1998; van de Kaa, 2008) or maybe even to six (Myrskyla, Kohler & Billari, 2009).

The theory was proposed in its inchoate form in 1929 (Thompson, 1929) but the word "transition" was not used until 1934 (Landry, 1934). The first clear outline of this theory is attributed to Notestein (1945). Its fundamental concepts illustrated in Figure 1 are based on the illustration prepared by Montgomery (n.d) and by van de Kaa (2001; 2002).

Stage 1 is claimed to have "prevailed since time immemorial" (Komlos, 2000, p. 320), i.e. for many thousands of years. The characteristic feature of this stage is the high birth and death rates fluctuating around the same constant value and producing a stagnant state of growth. The size of the population remained approximately constant and the rate of natural increase approximately zero. This stage is described as the Preindustrial Age, the Preindustrial Society, the Malthusian Regime, the Epoch of Malthusian Stagnation, the Pre-Demographic Transition Stage and the Age of Pestilence and Famine. Living conditions during that long time are claimed to have been characterised by poor health care, poor hygiene, "inadequate diets, as well as unsanitary drinking water and bacterial diseases" (Warf, 2010:708). During this stage, there was a continuing struggle for survival and the growth of the population was "fluctuating around zero" (Warf, 2010:708).

Stage 2 is supposed to have been dramatically different. It was the stage of population explosion, usually linked with the Industrial Revolution, the stage of transition from ages-long stagnation to a rapid growth of the population. The rate of natural increase is supposed to have started to increase rapidly and the size of the population exploded. This stage is described as the Early Industrial Society, the Early Industrial Age, the Post-Malthusian Regime, the Early-Demographic Transition and the Age of Receding Pandemics. The transition from Stage 1 to Stage 2 is described as the Escape from the Malthusian Trap, the Great Escape and as the population explosion.

The characteristic feature of this stage is supposed to have been the rapidly declining death rate described as the *mortality transition*, presumably caused by the generally improving living conditions reflected in a substantially better health care, better hygiene, better access to clean water, improved sanitation and increased food production (Chrispeels & Sadava 1994; Galor & Weil, 2000; Thomlinson, 1965). These postulated new growth-promoting forces "ignited a population explosion" (McFalls, 2007). Another characteristic feature of this stage is the continuing high birth rate over a certain time followed by its gradual decline.

Stage 3 is the stage of the slowing down growth and is described as the Mature Industrial Age, the Late Industrial Society, the Modern Growth Regime, the Stage of the Late Demographic Transition, or the Stage of Degenerative and Man-made Diseases. The difference between the mechanism of growth in Stages 2 and 3, is explained by a change in personal preferences prompted by such factors as women joining work force, better education, the availability of contraceptives and by the general tendency to have smaller number of children in order to improve the standard of living.

Stage 4 is the stage of a stable size of the population and is described as the Post-industrial Society, the Post-industrial Age, the Age of Delayed Degenerative Diseases, the Post-Demographic Transition Stage or the Stage of Invincibility. This stage is characterised by a close balance between birth and death rates, similar to the balance claimed for the Stage 1, but now both rates are low. Low birth rate is explained by personal preferences of replacing quantity by quality. The impact of infectious diseases during this stage is claimed to be low and to be replaced by harmful changes in the lifestyle. Mortality is now "associated with smoking and obesity, as well as, to a lesser extent, car accidents, suicides, and homicides" (Warf, 2010:710).

We shall now examine the data for Sweden, Mauritius and England and we shall show that they are in contradiction with the Demographic Transition Theory, but in perfect harmony with other contradicting evidence (Nielsen, 2013; 2016; von Foerster, Mora & Amiot, 1960). The evidence is already strong. Demographic Transition Theory has no place in science.

In comparing this theory with empirical evidence it is essential to understand the characteristic features of the supposed Stage 1 and of the transition to Stage 2.

1. The supposed Stage 1 should be characterised by strong fluctuations in birth and death rates.
2. On average, birth and death rates should be high and nearly constant.
3. The gap between the fluctuating birth and death rates during this first stage should be on average zero.
4. There should be convincing evidence of stagnation in the growth of the population during the supposed Stage 1.
5. There should be a clear and convincing transition from Stage 1 to Stage 2, marked by a clear change in the pattern of growth of human population, *from stagnation to growth*, so clear that it could be described as a takeoff, spurt, or explosion.
6. The transition should be marked by a clear change in the pattern of birth and death rates. On average, death rates should start to decrease, while the birth rates should, for a certain limited time, remain constant and then they should also start to decrease.
7. The gap between birth and death rates should be progressively getting wider from approximately zero to a certain maximum value, which would mark the beginning of Stage 3.

Examination of data for Sweden

The data for Sweden (Statistics Sweden, 1999), used repeatedly in support of the Demographic Transition Theory, are displayed in Figure 2.

These data appear to be in support of the four stages of growth (*cf* Figure 1): Stage 1 characterised by large, nearly constant and strongly fluctuating birth and death rates; Stage 2 characterised by a widening gap between the average values of birth and death rates; Stage 3 characterised by a decreasing difference between the birth and death rates; and Stage 4 characterised by low and nearly equal birth and death rates. These data show also the gradually decreasing fluctuations in birth and death rates.

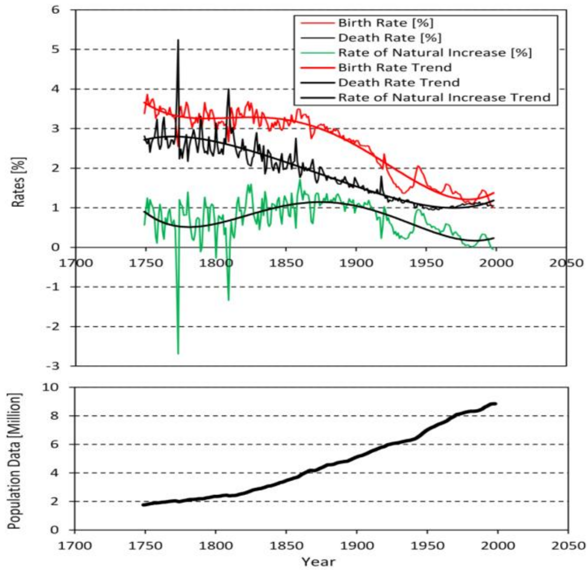

Figure 2. *Demographic Transition Theory is contradicted by the data for Sweden (Statistics Sweden, 1999). The four stages of growth did not exist.*

There was no stagnation and no explosion, takeoff or spurt in the growth of the population. The data also show that even large fluctuations in birth and death rates have no impact on the growth of human population.

However, what should notice immediately is that birth and death rates in the supposed Stage 1 do not fluctuate around the same constant value.

In order to produce a stagnant state of growth, birth and death rates have to vary around the same constant value. It is essential for the difference between them to be on average zero. It could vary between negative or positive values but it should not be on average larger than zero.

Data for Sweden should have never been used to illustrate the Demographic Transition Theory. Likewise, data for death rates *or* birth rates should never be used to test the Demographic Transition Theory. They should be used *together* because the Demographic Transition Theory describes how *both of them* should behave.

In the same source (Statistics Sweden, 1999) there are also aggregate data describing the growth of the population in Sweden, shown in the lower section of Figure 2. These data clearly demonstrate that the four stages of growth did not exist. They should have never been ignored in testing the Demographic Transition Theory.

Data for Sweden should have never been used to illustrate the validity of the Demographic Transition Theory because such illustrations are incorrect and misleading. When used in classrooms or lecture rooms, they do not teach science. When used in academic publications in support of the Demographic Transition Theory they propagate unscientific and incorrect information.

Death rate shown in the upper section of Figure 2 is decreasing in the apparent agreement with Stage 2 but it was also decreasing in the apparent Stage 1. There was no clear mortality transition, which could be claimed as marking the change from Stage 1 to Stage 2. Consequently, the apparent Stage 2 cannot be identified as Stage 2, which puts in questions other apparent stages.

The widening gap during the apparent Stage 2 is only slightly larger than the gap during the apparent Stage 1. Such a small change could not have produced a desired transition from a stagnant growth during the supposed Stage 1 to an explosive growth during the apparent Stage 2. In fact, the wide gap between birth and death rates during the supposed Stage 1 is obviously so large that there must have been no stagnation during this stage but a steadily-increasing growth of the population, and indeed this expectation is confirmed by the aggregate data describing the growth of the population in Sweden and shown in the lower part of Figure 2.

The disagreement between data and the Demographic Transition Theory is made even clearer if we look at the growth of human population in Sweden. They show clearly that *the four stages of growth did not exist*. Demographic Transition Theory neither describes nor explains the growth of human population in Sweden and is in gross disagreement with data

The data also show that even violent fluctuations in birth and death rates and the resulting fluctuations in the rate of natural increase have no impact on the growth of human population. The fluctuations in birth and death rates did not produce the normally expected Malthusian oscillations in the growth of human population.

A study of such fluctuations might be interesting for another reason but it has no bearing on explaining the mechanism of growth of human population. If we look at Figure 2, we can see that some points for the rate of natural increase are located far from the prevailing trend and yet even such large fluctuation had no noticeable effect on the recorded size of the population.

Summary of the contradicting evidence:

1. Contrary to the Demographic Transition Theory, the gap between birth and death rates during the supposed Stage 1 is not close to zero.

2. Such a wide gap cannot produce a stagnant state of growth characterised by a zero rate of natural increase, and indeed the data show that the rate of natural increase during this supposed Stage 1 was not zero.

3. The gap between birth and death rates during the supposed Stage 2 is only slightly larger than during the supposed Stage 1.

4. Such a difference is the size of the gap cannot produce the population explosion, and indeed there was no population explosion in Sweden during the displayed time.

5. "Mortality transition" (the decreasing death rate) commenced during the supposed Stage 1 and consequently, the supposed Stage 1 is not Stage 1.

6. Population data demonstrate that the four stages of growth did not exist. They show that there was a steadily increasing growth of the population.

7. Demographic Transition Theory is contradicted by the data for Sweden.

Examination of data for Mauritius

Data for Mauritius (Lehmeyer, 2004; Mauritius, 2015; Statistics Mauritius, 2014; UN, 2013) are shown in Figure 3.

Using the data for Mauritius in support of the Demographic Transition Theory (e.g. Lutz & Qiang, 2002) is surprising, because the population in Mauritius represents a minute fraction of the world population, and thus these data can be hardly considered as representing typical patterns of birth and death rates. Furthermore, these data are poorly documented and it is uncertain, which areas were included in the population surveys.

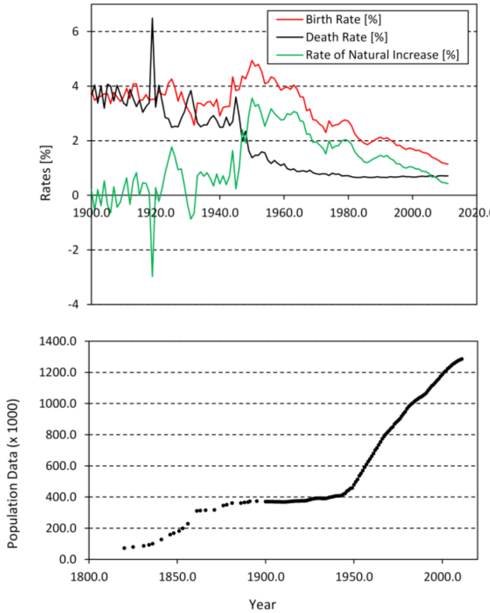

Figure 3. *Demographic Transition Theory is contradicted by the data for Mauritius (Lehmeyer, 2004; Mauritius, 2015; Statistics Mauritius, 2014; UN, 2013).*

The apparent Stage 1 is not Stage 1 because it was preceded by a fast growth of the population. Please notice that the time scales for the two diagrams are not the same.

The data describing birth and death rates, shown in the upper section of Figure 3, appear to be supporting the Demographic Transition Theory. Birth and death rates are at first high and they appear to be fluctuating around the same constant value, suggesting Stage 1 of growth.

There is also a clear mortality transition at a certain time marked by the rapidly decreasing death rate, accompanied by an increasing gap between birth and death rates, in good agreement with the pattern expected for Stage 2, characterised by a transition from stagnation to an explosive growth of the population. Gradually, the gap between birth and death rates narrows suggesting Stage 3 with a possibility of developing into Stage 4.

However, this apparent agreement with the theory becomes questionable when we look at the time scale. The "epoch of stagnation" as indicated by the merging birth and death rates lasted for only around 20 years. We could, perhaps, extend it to 40 years but we can see that the gap between birth rates started to increase from around 1920. The "epoch" is probably nothing more than a temporary delay in the growth of the population.

One of the fundamental principles of scientific investigation is that no relevant data should be ignored. Consequently, in order to understand the patterns displayed by birth and death rates we have to include also the data describing the growth of the population. These data are shown in the lower part of Figure 3 and they now make it perfectly clear that they do not support the Demographic Transition Theory, because the population was increasing before the apparent Stage 1. Consequently, the apparent Stage 1 is not Stage 1, which means that the apparent Stage 2 is not Stage 2. The whole pattern of growth is incompatible with the Demographic Transition Theory. The growth of the population in Mauritius was increasing, sometimes faster and sometimes slower, in complete disagreement with the Demographic Transition Theory.

The data show that over the displayed time the growth of the population was at first slow, then fast, slowing down, slow, fast, and slowing down again. Data for Mauritius demonstrate that there were more demographic transitions than claimed bythe Demographic Transition Theory.

Summary of the contradicting evidence:

1. While the gap between birth and death rates is close to zero as required by the Demographic Transition Theory for the Stage 1, the empirical evidence indicates that the stagnant state of growth lasted for only about 20 years or at best for only 40 years. The required evidence should be for at least a few hundred years, but in principle it should be for thousands of years.
2. The apparent Stage 2 looks like Stage 2 but this interpretation is contradicted by the population data showing that a similar stage of a fast growth was before the apparent Stage 1
3. Population data show that there were three, maybe even four, stages of growth during the displayed short time but these stages have nothing to do with the Demographic Transition Theory.
4. Demographic Transition Theory is contradicted by the data for Mauritius.

Examination of data for England

Probably the best, the most reliable and the most extensive demographic data we might ever expect to have are for England (Wrigley & Schofield, 1981) between 1541 and 1871. These data are important not only because of their high accuracy but also because they extend into the time well before of the Industrial Revolution, dated between 1760 and 1840 (Floud & McCloskey, 1994). Furthermore, it is important that these data are for England, where the impacts of the Industrial Revolution on the growth of human population should be strong and clear.

It is here, in England, that we should expect a clear confirmation of a change from high birth and death rates fluctuating around the same constant value to a new pattern characterised by a rapidly widening gap between these two quantities, indicating a clear transition from stagnation to population explosion. It is here, in England, that we should be able to see a clear *correlation* between the Industrial Revolution and the morality transition (the decreasing death rate); the clear confirmation of the beneficial effects of modern progress; the clear evidence of a dramatic escape from the Malthusian Trap; the dramatic transition from Malthusian stagnation (marked by a stagnant stage of growth characterised by Malthusian oscillations) to a rapid and sustained growth of human population.

Birth and death rates, together with the corresponding rate of natural increase in England are shown in Figure 4. The time-dependent patterns are entirely different than claimed by the Demographic Transition Theory (*cf* Figure 1).

Birth and death rates were always high – before, during and after the Industrial Revolution. They were also not fluctuating around a common constant value before the Industrial Revolution and there was no morality transition coinciding with this event. In fact, Industrial Revolution had no impact on the time-dependent distributions of birth and death rates. It is as if this crucial development, which was supposed to have had such a dramatic impact on the growth of human population had never happened.

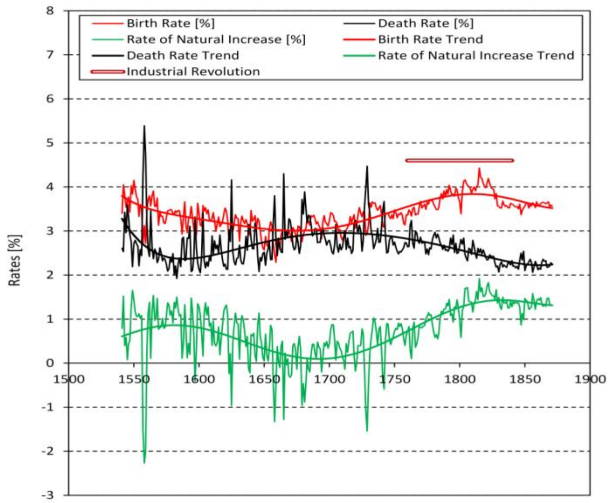

Figure 4. *Demographic Transition Theory is contradicted by the data for England (Wrigley & Schofield, 1981).*

The patterns claimed for the first two stages of growth are absent. Demographic Transition Theory presents a completely different story than the data.

The data show not just one but two mortality transitions (decreasing mortality rate) both beginning well before the commencement of the Industrial Revolution. Correspondingly, the data show not just one but two maxima in the rate of natural increase. The increase in the rate of natural increase leading to these two maxima began well before the Industrial Revolution. This increase was clearly *not* caused by the Industrial Revolution. A delayed response to the benefits of progress associated with the Industrial Revolution could be easily explained, but it would be hard, if not impossible, to explain the *anticipated* response.

The growth of human population in England between 1541 and 1871 (Wrigley & Schofield, 1981) is shown in Figure 5. The top panel shows all the data at yearly intervals. The lower panel shows data at larger time interval to allow for comparing them with the numerical integration of the fluctuating rates of natural increase.

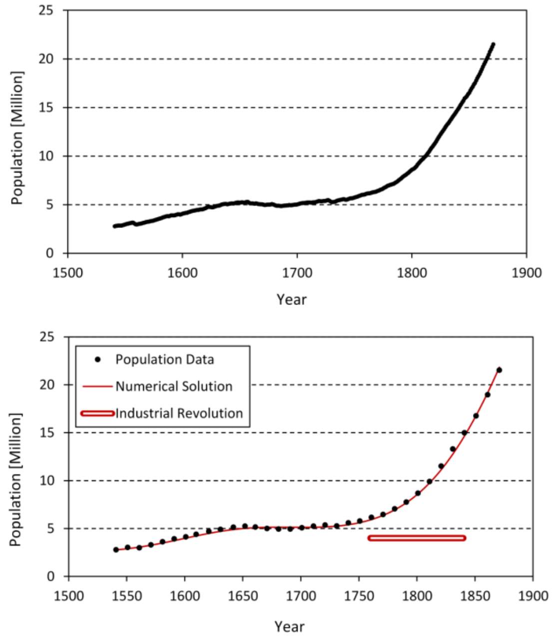

Figure 5. *Growth of human population in England (Wrigley & Schofield, 1981).*

Fluctuations in birth and death rates shown in Figure 4 had no impact on the growth of the population. The typical pattern of stagnation followed by explosion is not confirmed by data. The population was increasing well before the onset of the Industrial Revolution. After a short delay, the population started to increase again but the onset of this new growth was also before the Industrial Revolution.

The data for birth and death rates and for the population, shown in Figures 4 and in the upper panel of Figure 5, respectively, are at yearly intervals. However, while the data for birth and death rates and for the corresponding rate of natural increase show strong fluctuations, the data for the growth of the population does not show even a slightest effect of these fluctuations. *The growth of the population is immune to the fluctuations in birth and death rates.*

"These models of Malthusian oscillations, although elegant and intriguing, must be viewed as quite speculative in their application to any actual populations" (Lee, 1997). Indeed, their presence is contradicted by the data for England, Sweden and Mauritius as well as by the analyses of the world population data (Nielsen, 2013; 2016).

We have demonstrated that the fluctuations in birth and death rates are not reflected in the growth of human population. However, we can reverse our investigation and ask whether the fluctuations in birth and death rates can generate fluctuations in the *calculated* growth of human population. Suppose we use the empirically-determined birth and death rates or the rate of the natural increase representing the difference between the birth and death rates, and suppose that we use these rates to *calculate* the size of the population, will they produce the fluctuations *in the calculated distribution*?

In order to answer this question, we have carried out numerical integration of the following differential equation:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = R_e(t) \tag{1}$$

where $S(t)$ is the *calculated* size of human population and $R_e(t)$ is the empirically-determined, and *fluctuating*, rate of natural increase shown in Figure 4 and calculated using the empirically-determined, and *fluctuating*, birth and death rates. Migration rates are relatively small (Wrigley & Schofield, 1981) and can be neglected. However, if the calculated distribution of the size of the population is not going to agree with population data, they will have to be included.

Results of these numerical calculations are shown in the lower part of Figure 5. The calculated curve is displayed in steps of one year but it follows that data so closely that in order to see any possible fluctuations we had to show data at 10-year intervals. The fluctuating birth and death rates or the corresponding fluctuating rate of natural increase do not produce even the slightest fluctuations in the calculated distribution describing the growth of the population.

The growth of the population shown in Figure 5 does not display the expected pattern of stagnation followed by explosion claimed by the Demographic Transition Theory. There was a steady growth of the population well before the Industrial Revolution. This growth was briefly interrupted but it was resumed again around 1700. The growth of the population in England is not correlated with the Industrial Revolution. There is no indication of prolonged Malthusian stagnation, no evidence of Malthusian oscillations, no clear evidence of the existence of stage one and no transition to a new stage. It is just a growth, which was increasing, halted for a while and started to increase again. Demographic Transition Theory is contradicted by data.

The growth of human population can be also studied using the reciprocal values of data, $1/S(t)$. Such a study gives a new insight into the interpretation of data. This method has been discussed elsewhere (Nielsen, 2014).

Reciprocal values of the size of human population in England and their absolute gradient, calculated directly from data and interpolated, are shown in Figure 6.

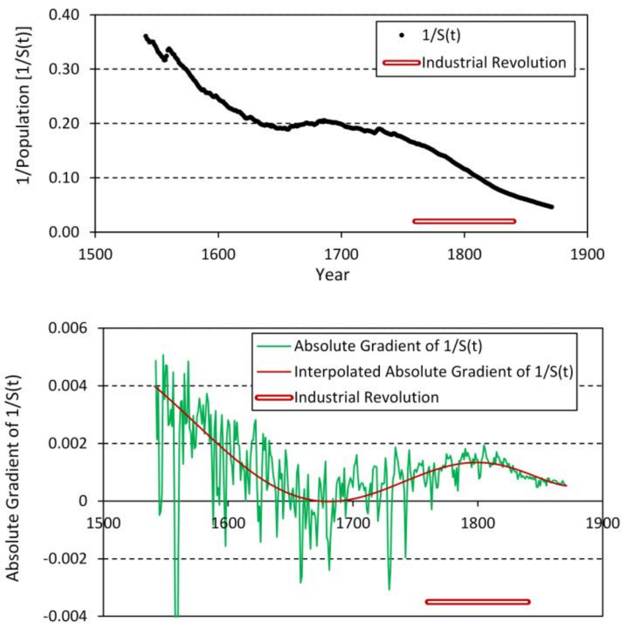

Figure 6. *Reciprocal values,* $1/S(t)$ *, of the size of the population and their absolute gradient calculated directly from data and interpolated.*

There was no stagnation before the Industrial Revolution and no boosting of growth by the Industrial Revolution. On the contrary, the Industrial Revolution coincides with the slowing-down growth as indicated by the maximum in the interpolated gradient.

The deceasing reciprocal values, $1/S(t)$, of the size of the population indicate an increasing growth, and *vice versa*. The top section of Figure 6 shows that the population in England was steadily increasing well before the Industrial Revolution, as indicated by the steadily-decreasing reciprocal values. After only a brief interruption, the size of the population in England continued to increase, confirming the pattern of growth shown in Figure 5.

There was no stagnation that could be identified as Stage 1 proposed by the Demographic Transition Theory. This stage did not exist. In its place there was, in general, a steadily-increasing growth. The data show a temporary distortion of this trajectory but the general pattern was a continuing increase of the population. Furthermore, the data show no correlation of the growth of the population with the Industrial Revolution. There was no transition to a distinctly new stage. This pattern of growth is in contradiction with the pattern proposed by the Demographic Transition Theory.

The absolute gradient of the reciprocal values, $1/S(t)$, of the size of the population is also a convenient indicator allowing for detecting whether the growth was accelerating or decelerating. The decreasing absolute gradient indicates a slowing-down growth while the increasing gradient indicates an acceleration.

The absolute gradient of the $1/S(t)$ data is shown in the lower part of Figure 6. If we compare the upper and the lower sections of this figure we can see that the growth of the population in England was steadily increasing, as indicated by the decreasing reciprocal values $1/S(t)$, but it was gradually getting slower, as indicated by the decreasing absolute gradient of the reciprocal values. After a short period of instability, the growth of human population in England started to increase again from around 1690 and was accelerating, as indicated by the downward bending of the reciprocal trajectory and by its increasing absolute gradient. The onset of this new growth occurred about 70 years before the onset of the Industrial Revolution. Contrary to the general beliefs, *the Industrial Revolution did not boost the growth of human population in England* where its impacts should be stronger than anywhere else.

After a certain time, the acceleration of the growth of human population started to grow weaker, as indicated by the gradient approaching its maximum value. The absolute gradient of the reciprocal values reached its maximum around 1800 or right in the middle of the Industrial Revolution and then started to decrease. The growth of human population started to decelerate.

If we wanted to claim a cause-effect link between the Industrial Revolution and the growth of human population in England we could conclude that the Industrial Revolutions slowed down the growth of the population and diverted it to a slower trend as indicated by the decreasing absolute gradient of the reciprocal values. However, more plausible conclusion is that the *Industrial Revolution had no impact on the growth of human population*. The two processes were totally independent and it is incorrect to link them by any cause-effect properties. The growth of human population in England must have been prompted by different forces than the forces associated with the Industrial Revolution and with the numerous random forces repeatedly proposed to explain the epoch of stagnation, which did not exist.

Summary of the contradicting evidence:

1. The time dependence of birth and death rates in England between 1541 and 1871 (Wrigley & Schofield, 1981) is in contradiction with the first two stages of growth claimed by the Demographic Transition Theory. The pattern of the fluctuating birth and death rates around a common high constant value followed by a clear transition to a new stage around the time of the Industrial Revolution is contradicted by data.
2. The first stage of growth proposed by the Demographic Transition Theory did not exist.
3. The data show not just one mortality transition (decreasing death rate) as claimed by the Demographic Transition Theory, but two, both of them beginning well before the onset of the Industrial Revolution.
4. There is no positive correlation between the Industrial Revolution and the time dependence of birth and death rates.
5. Data for England show that there were more demographic transitions than can be accounted for by the Demographic Transition Theory.
6. With the exception of a minor delay between around 1656 and 1682, the growth of human population in England was steadily increasing.
7. Reciprocal values of data for the size of human population also confirm that Industrial Revolution had no impact on the growth of the population in England, where it should have been stronger than anywhere else
8. Rather than being boosted by the Industrial Revolution, the growth of the population in England started to be diverted to a slower trajectory from around 1800.
9. Demographic Transition Theory is contradicted by the data for England between 1541 and 1871.

Summary and conclusions

Data for Sweden, used repeatedly in support of the Demographic Transition Theory, are shown to be in its direct contradiction. They show that the four stages of growth claimed by the Demographic Transition Theory did not exist. There was no stagnation (no Stage 1) and no transition to a new stage (Stage 2) claimed by the Demographic Transition Theory. There was no population explosion and no transitions to stages three and four. There was just a steadily-increasing, single-stage, growth of the population. The gap between death and birth rates in the apparent Stage 1 was large and there was no dramatic change in its size during the usually claimed but non-existent transition from Stage 1 to Stage 2.

The data for Mauritius, used sometimes in support of the Demographic Transition Theory (e.g. Lutz & Qiang, 2002) also show a clear disagreement with this theory. The apparent Stage 1 suggested by the birth and death rates, even if accepted, lasted for only a few decades. However, when aggregate data are included, they show that the apparent Stage 1 was not Stage 1 because it was not preceded by stagnation but by a steadily increasing growth of the population. The pattern of growth of human population does not fit into the pattern claimed by the Demographic Transition Theory

The exceptionally good data for England, 1541-1871, are also in contradiction with the Demographic Transition Theory. The expected stages in birth and death rates are not confirmed by the data. There were two mortality transitions during that time, both commencing well before the onset of the Industrial Revolution. There is no correlation between the Industrial Revolution and the time-dependence of the birth and death rates in England. There was no stagnation followed by population explosion.

Industrial Revolution did not boost the growth of human population in England. On the contrary, the data show that from around 1800 the growth of the population in England started to be slowing down. Consequently, if we want to link the Industrial Revolution with the growth of the population, we would have to conclude that the Industrial Revolution slowed down the growth of the population. However, more plausible conclusion is that the two processes were totally independent. The data indicate that it is incorrect to use the Industrial Revolution to explain the mechanism of growth of human population, even in England, the centre of this revolution.

While the data for Sweden show a steady growth of the population without any signs of four stages claimed by the Demographic Transition Theory, the data for Mauritius and England demonstrate that there were more stages than one can account for by using the Demographic Transition Theory.

The study presented here also shows that even large fluctuations in birth and death rates have no impact on the growth of human population. It is, therefore, incorrect to imagine that fluctuations in birth and death rates can produce Malthusian oscillations in the size of the population. Furthermore, moderate variations in the growth rate or the rate of natural increase can, at best, create only small and negligible variations in the growth of population.

A study published over 50 years ago (von Foerster, Mora & Amiot, 1960) demonstrated that the growth of human population was hyperbolic. In science, even one contradicting evidence is sufficient to show that a contradicted theory is incorrect. Now we have more extensive sources of data (Maddison, 2010; Manning, 2008; US Bureau of Census, 2015). They all show that there was no stagnation in the growth of human population (Nielsen, 2013; 2016). They show clearly that the Stage 1 claimed by the Demographic Transition Theory did not exist and that there was no transition to the supposed Stage 2. They show that the Demographic Transition Theory is contradicted by data.

Demographic Transition Theory has a strong link with the Unified Growth Theory (Galor, 2005; 2011), which also claims, incorrectly, the existence of the epoch of stagnation and a dramatic transition to a new stage of economic growth described repeatedly as takeoff. A study of the income per capita (GDP/cap) combines the study of the economic growth, as expressed by the GDP, and the study of the growth of the population.

The time distribution of the historical GDP/cap values is claimed in the Unified Growth Theory to be made of a prolonged stagnation followed by a sudden takeoff in much the same way as the Demographic Transition Theory claims that the growth of human population can be represented by a prolonged stage of stagnation followed by a sudden explosion. Both interpretations are incorrect and both of them are based by illusions reinforced by the incorrect interpretations of hyperbolic growth.

The growth of the population was slow over a long time and fast over a short time but it was slow because it was hyperbolic and fast because it was hyperbolic. It was a monotonically-increasing hyperbolic distribution (Nielsen, 2016; von Foerster, Mora & Amiot, 1960). Economic growth, whether expressed in terms of the GDP or GDP/cap, was slow over a long time and fast over a short time but it was slow because it was hyperbolic and fast because it was hyperbolic (Nielsen, 2014; 2015). There was no stagnation and no sudden takeoff.

Demographic Transition Theory is incorrect and the only way to accept it is by ignoring the repeatedly contradicting empirical evidence and by placing full trust in stories based largely on creative imagination perfected by accretion over many years and by many people, each new imagined explanation or concept creating new ideas and all growing into the established knowledge in demography.

The Demographic Transition Theory (or Model) has been a ghost story for at least 20 years (Abernethy, 1995) and it is not clear why its concepts have not been abandoned long time ago. It would be probably better to accept openly and clearly that each case should be studied individually and that it is not necessary to reconcile them with some kind of a master theory, which at present does not exist.

Scientific principles of investigation can be used even in the absence of an all-encompassing theory, and the fundamental principle is to refrain from ignoring any relevant data particularly if they contradict the accepted interpretations. It appears that the continuing use of the Demographic Transition Theory makes the demographic research unscientific because by now and over many years this field of research evolved into a strong system of concepts many of which can be accepted only by faith.

There is also another serious problem with the continuing toleration of this theory. Demographers might be aware of the fundamental problems associated with Demographic Transition Theory. However, many teachers, lecturers and university professors might be less informed. They accept it as scientific and they teach it to younger generations, who accept this theory as presented to them believing that they learn science.

For instance, quite recently, Thompson & Roberge (2015) published an article in which they present a diagram showing the four stages of growth proposed by the Demographic Transition Theory. They show how to help students to unpack "this rich display of information" (p. 254) without being aware that they are helping to unpack this rich source of *misinformation*. It would be more useful to teach students why the diagram they see is a misleading source of misinformation. It is a fiction story, a ghost story, presented as science, but teachers might not be aware of the problems permeating the corridors of science.

Correct understanding of the growth of human population is important but the misleading information presented by the Demographic Transition Theory is seriously harmful because this theory does not explain the growth of human population but presents concepts and explanations, which when closely examined are contradicted by empirical evidence. It is better to have no theory than a misleading theory.

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6. Growth of the world population in the past 12,000 years

Introduction

The study of the historical economic growth involves not only the study of the Gross Domestic Product (GDP) but also the study of the growth of the population, because as pointed out by Galor (2005; 2011), it is important to understand the relationship between these two process and particularly the relationship between the growth of the income per capita (GDP/cap) and the growth of the population. It is, perhaps, for this reason that the latest and the most extensive compilation of the historical GDP data, published by the world-renown economist, includes also the data describing the historical growth of human population (Maddison, 2001; 2010).

About 50 years ago, von Foerster, Mora & Amiot (1960) demonstrated that human population was increasing hyperbolically during the AD era. We now have far better and more extensive sets of data compiled not only by Maddison (2001; 2010) but also by Manning (2008) and by the US Census Bureau (2016). The last two compilations are based on virtually the same primary sources but they are complimentary.

Maddison's compilation is useful in studying the growth of the population not only global but also regional and national. However, his data are terminated in AD 1. Furthermore, they also contain significant gaps below AD 1500. The data compiled by Manning and by the US Census Bureau are significantly richer but they are limited only to the description of the world population. However, they extend down to 10,000 BC.

It is well outside the scope of the discussion presented here, but a preliminary examination of Maddison's data indicates that the economic growth and the growth of human population followed similar trajectories. Consequently, by using a rich set of data extending down to 10,000 BC we might gain a better insight not only into the historical growth of human population but also to its possible link with the economic growth.

The data

Procedures adopted in estimating historical populations are described by Durand (1977). The data for the AD era are of exceptionally good quality. Between AD 400 and 1850, independent estimates are within ±10% of their corresponding averaged values. The estimates after 1850 are within ±1.5%. The largest deviations of around ±30% are for the AD 1 data. The two estimates for AD 200 differ by ±15% from their average value. The BC data are less accurate and less consistent but when closely analysed they are also found to follow a certain, well-described trajectory.

Analysis of population data

In order to understand hyperbolic distributions, it is useful to compare them with the more familiar exponential distributions. The differential equation describing exponential growth is given by the following simple equation:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = k\,, \qquad (1)$$

where $S(t)$ is the size of a growing entity, in our case the size of the population, and k is an arbitrary constant.

The left-hand side of this equation represents growth rate. For $k > 0$ the eqn (1) describes growth, while for $k < 0$ it describes decay.

The solution of the eqn (1) is

$$S(t) = ae^{kt}\,, \qquad (2)$$

where a is the constant related to the constant of integration.

The eqn (2) gives

$$\ln S(t) = \ln a + kt \, . \tag{3}$$

The logarithm of the size of the growing entity increases linearly with time. Exponential growth can be easily identified by plotting data using semilogarithmic scales of reference because in such presentation the data should follow an increasing straight line.

Data for the growth of the population during the BC and AD eras (Manning, 2008; US Census Bureau, 2015) are shown in Figure 1. They are compared with the best exponential fit to the data. The world population was not increasing exponentially.

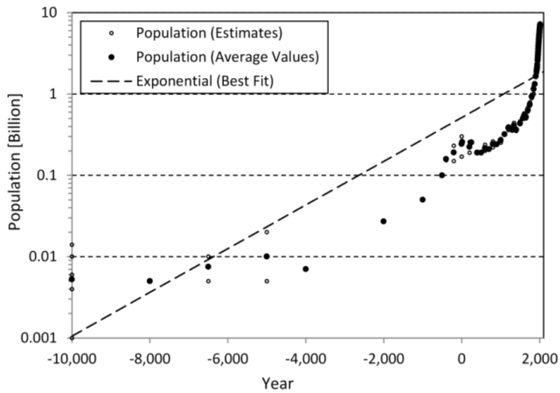

Figure 1. *Data describing the growth of the world population (Manning, 2008; US Census Bureau, 2016) are compared with the best fit using exponential function. The world population was not increasing exponentially. The BC time scale is identified by the negative numbers.*

Let us now examine the hyperbolic growth. This type of growth is described by the following differential equation:

$$\frac{1}{S(t)} \frac{dS(t)}{dt} = kS(t) \, , \tag{4}$$

where $k > 0$.

It is a slight modification of the eqn (1). Here, the growth rate is not constant but directly proportional to the size of the growing entity. The solution of this equation, which can be found by substitution $S(t) = Z^{-1}(t)$, is given by the following simple formula:

$$S(t) = \frac{1}{a - kt}. \tag{5}$$

It is just a reciprocal of a linearly-decreasing function. Consequently,

$$\frac{1}{S(t)} = a - kt \tag{6}$$

The reciprocal values of the size of the growing entity follow a decreasing straight line. This representation simplifies the analysis of hyperbolic distributions. We can use this dependence to identify uniquely hyperbolic growth, in much the same way as the linearly increasing logarithm of the growing entity can be used to identify exponential growth.

It is now useful to understand the difference between the exponential growth and the hyperbolic growth. For the exponential growth, the growth rate is constant. It does not matter how large is the size of the growing entity, the growth rate never changes. For this reason, exponential growth can be characterised and identified by using the growth rate or equivalently by using the doubling time. This approach is inapplicable to the hyperbolic growth or to any other type of growth, for that matter. That is why it is incorrect to use the doubling time to characterise any other type of growth. In particular, it is incorrect to use the so-called "rule of 70" for any other type of growth because in all other cases the growth rate and the doubling time are not constant. In order to characterise any other types of growth by the growth rate or by the doubling time we cannot just present a single value for any of these two quantities at a certain time but we have to show how their growth rate or the doubling time depends on time or on the size of the growing entity. For instance if we look at the eqn (4) we can see that, for the hyperbolic growth, the growth rate is *directly* proportional to the size of the growing entity. This is a useful characteristic feature of hyperbolic growth. Another characteristic feature of hyperbolic growth is that the growth rate *per size* of the growing entity is constant.

As discussed elsewhere (Nielsen, 2014), analysis and interpretation of hyperbolic distributions is difficult because they appear to be made of two distinctly-different components, slow and fast, leading to countless misconceptions and misinterpretations of hyperbolic distributions describing the growth of human population or the economic growth. However, the analysis of these distributions and their interpretation become trivially simple if the reciprocal values are used, as shown in Figure 2, because according to the eqns 5 and 6, if data follow a decreasing straight line, then the growth is hyperbolic. We can then fit the reciprocal values to find the mathematical expression for the hyperbolic growth given by the eqn (5).

Furthermore, if the reciprocal values of data follow a decreasing straight line, the growth is not stagnant but hyperbolic. However, the concept of stagnation is not supported even if the reciprocal values of data do not decrease linearly. Any monotonically-decreasing trajectory will show that the postulate of stagnation followed by a takeoff at the certain time is not supported by data. To prove the existence of the epoch of stagnation it is necessary to prove the presence of random fluctuations often described as Malthusian oscillations. Such random fluctuations should be clearly seen not only in the direct display of data but also in the display of their reciprocal values. It they are absent then there is no support in data for claiming the existence of the epoch of stagnation. However, if the reciprocal values of data follow a decreasing straight line, then they show, or at least strongly suggest, that the growth was hyperbolic. Positive identification of any type of growth depends on the range of available data.

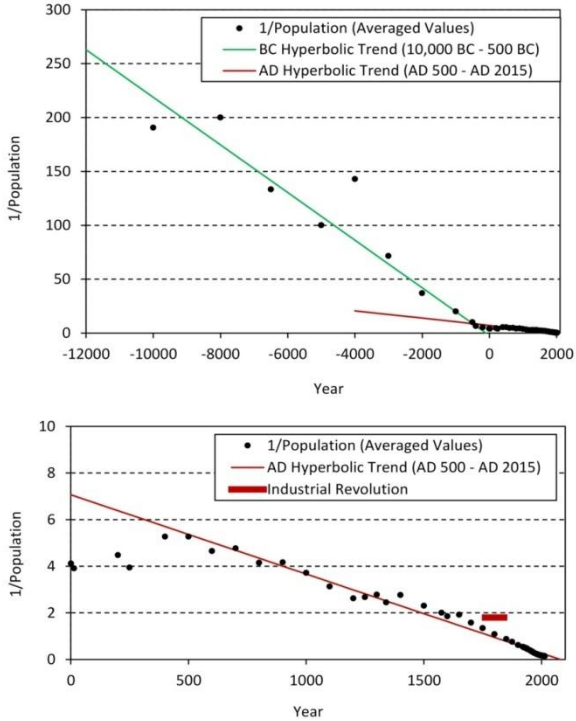

Figure 2. *Reciprocal values of the world population data (Manning, 2008; US Census Bureau, 2015) reveal two distinctly different hyperbolic trajectories (represented by the decreasing straight lines). They also show a dramatic demographic transition between around 500 BC and AD 500.Furthermore, they show that there was no takeoff around the time of the Industrial Revolution. In fact, there was no transition from stagnation to growth at any time. The size of the population is in billions.*

It should be also remembered that for the reciprocal values, the effects are reversed. A diversion to a *slower* trajectory will be indicated by an *upward* bending away from the earlier trajectory, while diversion to a *faster* trajectory will be indicated by the *downward* bending. Descriptions of the economic growth involve frequent discussions of the so-called takeoffs (Galor, 2005, 2011) representing the assumed sudden and prominent change in the growth trajectory, a transition from the supposed stagnation to growth. For the economic growth or for the growth of human population represented by their reciprocal values, such sudden takeoff should be indicated by a clear and strong downward bending of the growth trajectory.

If the straight line representing the reciprocal values of data remains unchanged, then obviously there is no change in the mechanism of growth. It makes no sense to divide a straight line into two or three arbitrarily selected sections and claim different regimes of growth controlled by different mechanisms for these arbitrarily-selected sections.

The analysis of data presented in Figure 2 reveals two distinctly different hyperbolic trajectories for the BC and AD eras. They are represented by two distinctly different straight lines fitting the reciprocal values of population data. In this representation, the growth during the AD era is dwarfed by the growth during the BC era but this part can be better examined by looking at the lower section of Figure 2.

The corresponding hyperbolic distributions are shown in Figure 3. Figures 1 and 2 make it clear that the growth of human population was not exponential, as it was expected by Malthus (1798). The data and their analysis show that *if unchecked, population increases hyperbolically*. It shows that the growth of human population was increasing hyperbolically not only during the AD era, as observed by von Foerster, Mora and Amiot (1960), but also during the BC era. This analysis shows also that the Industrial Revolution, 1760-1840 (Floud & McCloskey, 1994) did not boost the growth of human population, the result being in agreement with the analysis of the historical economic growth (Nielsen, 2016).

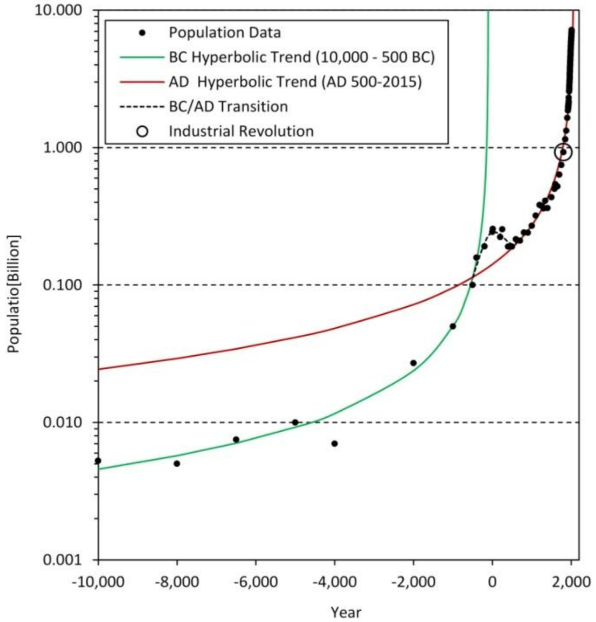

Figure 3. *If unchecked, population increases hyperbolically. This overall view shows that there was only one major demographic transition (between around 500 BC and AD 500) from a fast to a significantly slower hyperbolic trajectory. Industrial Revolution had no impact on the growth of human population. The perceived population explosion is the natural continuation of hyperbolic growth.*

Results presented in Figures 2 and 3 show that from 10,000 BC to around 500 BC the growth of human population was hyperbolic. This hyperbolic growth was followed by a demographic transition between 500 BC and AD 500 from a fast BC hyperbolic trajectory to a significantly slower AD hyperbolic trajectory. It was not a transition from stagnation to growth because there was no stagnation in the growth of human population (Nielsen, 2013a).

Hyperbolic parameters fitting the world population data are: $a = -2.282$ and $k = 2.210 \times 10^{-2}$ for the BC trajectory between 10,000 BC and 500 BC, and $a = 7.061$ and $k = 3.398 \times 10^{-3}$ for the AD trajectory between AD 500 and 2015.Characterised by the parameter *k*, the BC hyperbolic growth was 6.5 times faster than the AD growth.

Using the data (Manning, 2008; US Census Bureau, 2016), the fitted hyperbolic distributions (shown in Figure 3) and the eqn (4) we can now estimate the growth rate during the BC and AD eras.

During the BC era, the growth rate was increasing hyperbolically(monotonically) with time or linearly (and again monotonically) with the size of the population from around 1.010×10^{-4} (0.010%) per year in 10,000 BC to around 2.520×10^{-3} (0.252%) per year in 500 BC. The growth was slow but not stagnant. During the AD era, the growth was again approximately hyperbolic from AD 500 to 1950, and the growth rate increased approximately monotonically from 6.337×10^{-4} (0.063%, smaller than in 500 BC) and 7.805×10^{-3} (0.781%) in 1950.

There was also no stagnation but a hyperbolic growth. The transition between 500 BC and AD 500 was not a transition from stagnation to growth but from growth to growth. It was not a dramatic takeoff but a transition to a slower hyperbolic trajectory. These features are important in relating the growth of the population to the economic growth because contrary to the repeated claim in the Unified Growth Theory (Galor, 2005;2010) there was also no dramatic takeoff in the growth of the GDP (Nielsen, 2016) or in the growth of the GDP/cap (Nielsen, 2015a).

Detailed analysis of the AD data

The data for the AD era are of exceptionally good quality and they allow for a closer and minute examination of the pattern of growth. Even though the hyperbolic trajectory shown in Figures 2 and 3 fits the AD data well, the display of the reciprocal values presented in the lower part of Figure 2 shows that starting from around AD 1400, some data are systematically above the fitted straight line, suggesting a shift in the hyperbolic growth around that time.

Reciprocal values of data shown in Figure 4 reveal a clear delay in the growth of the population between around AD 1200 and 1400 followed by a new and slightly faster hyperbolic trajectory. Hyperbolic trajectory between AD 500 and 1200 is given by $a = 6.940$ and $k = 3.448\times10^{-3}$, and from AD 1400 by $a = 9.123$ and $k = 4.478\times10^{-3}$. For these new and improved fits to the data, growth rate was 6.610×10^{-4} (0.066%) in AD 500, 1.230×10^{-3} (0.123%) in AD 1200, 1.568×10^{-3} (0.157%) in AD 1400 and 1.142×10^{-2} (1.142%) in 1950. The growth was hyperbolic (monotonic) between AD 500 1200 and again between AD 1400 and 1950. There was no stagnation and no dramatic takeoff from stagnation to growth at any time.

Transition between AD 1200 and 1400 coincides with the unusual convergence of strong and lethal events, representing a *combined impact of five* significant demographic catastrophes (Nielsen, 2013b): Mongolian Conquest (1260-1295) with the total estimated death toll of 40 million; Great European Famine (1315-1318), 7.5 million; the 15-year Famine in China (1333-1348), 9 million; Black Death (1343-1352), 25 million; and the Fall of Yuan Dynasty (1351-1369), 7.5 million. This is the only evidence in the data that demographic catastrophes might have had influence on the growth of the world population and if such is the case, not one but five of them were need to generate a small distortion.

There is no indication that exogenous conditions after AD 1400 were different than before AD 1200 so the slightly faster hyperbolic growth from around AD 1400 could be explained by the natural human response to crisis manifested in the intensified process of regeneration (Malthus, 1798; Nielsen, 2013c).

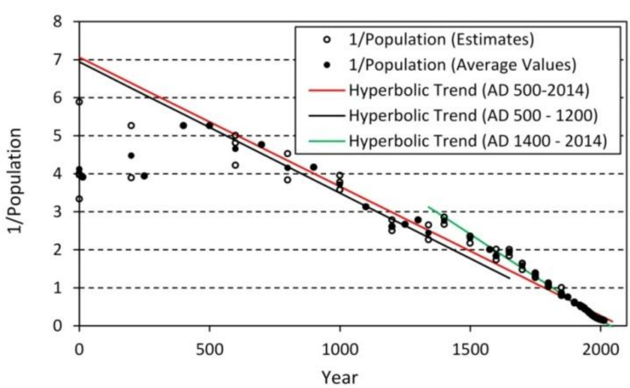

Figure 4. *Reciprocal values of data for the AD era show a clear but small disturbance in the growth of the population between AD 1200 and 1400. This disturbance caused a shift to a slightly faster hyperbolic trajectory. The size of the population in billions.*

Closer view of the new growth trajectory, starting from around AD 1400, is displayed in Figure 5. The new hyperbolic growth was undisturbed until around 1950 when it experienced a *small but unsustained* acceleration, as indicated by a slight downward bending of the trajectory of the reciprocal values. This minor boosting lasted for only a short time and soon the growth of human population started to be diverted to a slower trajectory, as indicated by the conversion of the temporary downward bending to upward bending of the trajectory of reciprocal values.

Again, there was no dramatic takeoff and no transition from stagnation to growth, the term used repeatedly by Galor (2005; 2011) and the feature, which was supposed to characterise not only the economic growth but also the growth of human population. The repeated claim of a dramatic transition (takeoff) from stagnation to growth is contradicted by the analysis of the economic growth (Nielsen, 2015a; 2016) and by the presented here analysis of the growth of the world population.

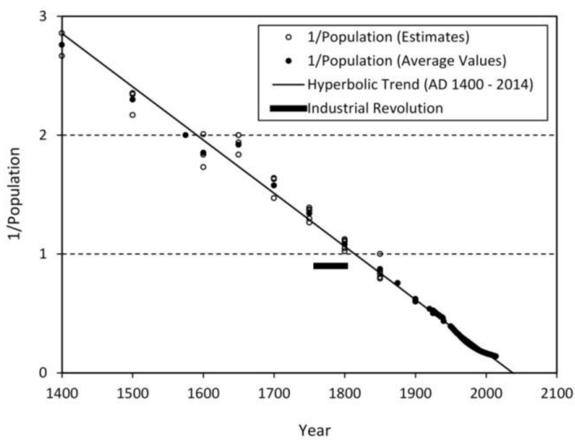

Figure 5. *Between AD 1400 and around 1950 the growth of human population was hyperbolic. Data show a minor boosting around 1950 followed quickly by a diversion to a slower trajectory. There was no takeoff from stagnation to growth at any time. Industrial Revolution had no impact on boosting the world population. The size of the population is in billions.*

It is remarkable that the growth of the world population was so hyperbolically stable over the past 12,000 years. The data show that during this long time that there were only three transitions: 500 BC - AD 500, AD 1200 - AD 1400 and 1950 - present. Each of the two earlier transitions was a shift between hyperbolic trajectories. The outcome of the current transition is unknown. The dynamics of growth in the past 12,000 years is summarised in Table 1.

Table 1. *Dynamics of growth of the world population in the past 12,000 years. Time intervals are approximate.*

Hyperbolic Growth	Demographic Transitions
10,000 BC – 500 BC $a = -2.282$; $k = 2.210\times10^{-2}$	500 BC – AD 500 Transition from a fast to much slower hyperbolic trajectory
AD 500 – 1200 $a = 6.940$; $k = 3.448\times10^{-3}$	AD 1200 – 1400 Transition from a slow to a slightly faster hyperbolic trajectory
AD 1400 – 1950 $a = 9.123$; $k = 4.478\times10^{-3}$	1950 – present Transition from a hyperbolic trajectory to an unknown trend
Total time of hyperbolic growth: 10,750 years (~89% of the total combined time)	Total rime of transitions 1265 years (~11% of the total combined time)

Implications for the economic growth

As mentioned earlier, preliminary analysis of Maddison's data (Maddison, 2001; 2010) shows close similarities between the distributions describing economic growth and the growth of human population. Galor also commented that there was a "positive relationship between income per capita and population that existed throughout most of human history" (Galor, 2005, p.177). The study of the economic growth goes hand in hand with the study of the growth of the population.

Our analysis demonstrated that the growth of the world population was hyperbolic, and consequently monotonic, and that there was never a transition from stagnation to growth, which could be described as a sudden takeoff. The fast-increasing growth of the world population in recent years was just the natural continuation of the hyperbolic growth.

Our analysis shows that with the exception of just two demographic transitions (500 BC - AD 500, and AD 1200 - 1400) the growth of human population was monotonic until around 1950, when it started to be diverted to a yet unknown trajectory. The first demographic transition (500 BC - AD 500) was from a faster to a slower hyperbolic growth. It was definitely not a takeoff from stagnation to growth. The second transition (AD 1200 – 1400) was from a slow to a slightly faster hyperbolic trajectory (only 30% faster, as indicated by the parameter *k*). It was also not a transition from stagnation to growth. The current transition, which commenced around 1950 was initially to a slightly faster trajectory, which was soon becoming progressively slower than the preceding hyperbolic trajectory. Here again, there was no transition from stagnation to growth. For 89% of the past 12,000 years the growth of human population was hyperbolic and monotonic and there was never a transition from stagnation to growth. Our analysis shows that the growth of human population was remarkably stable over the past 12,000 years.

Galor wonders "what is the origin of the sudden spurt in growth rates of output per capita and population?" (Galor, 2005, p. 177). This puzzle has now been solved: *there was no sudden spurt*.

Trying to explain this sudden spurt is like trying to explain why there is water in the middle of the desert when the image of water is created by a mirage. It is a waste of time and effort. We can explain the *illusion* of the spurt but not the spurt. The illusion of the spurt is explained by the hyperbolic properties but the sudden spurt has never happened. What we see as a sudden spurt is the natural continuation of the monotonically-increasing hyperbolic distribution and the simplest way to dispel the illusion of stagnation and of a sudden spurt is to use the reciprocal values of data (Nielsen, 2014) but we can also use other methods (Nielsen, 2015a). The point is that data have to be rigorously analysed. Any perfunctory and hasty examination of data is likely to lead to incorrect conclusions and we can find many examples of such examinations of data in the Unified Growth Theory (Galor, 2005; 2011).

We have demonstrated that there was no sudden spurt in the growth rate of the world population because the growth was hyperbolic, which means that the growth rate was also increasing hyperbolically with time or linearly with the size of the population, in both cases monotonically [see the eqn (4)]. Such an increase has no room for any form of spurts.

There were also no spurts during the past two demographic transitions. During the first transition (500 BC - AD 500), the growth rate decreased from 0.252% in 500 BC to 0.066% in AD 500. During the second transition (AD 1200 - 1400) the growth rate increased only slightly from 0.123% in AD 1200 to 0.157% in AD 1400.

So, our analysis eliminates at least one of Galor's spurts: the supposed spurt in the growth rate of human population. What remains to be explained is the supposed spurt in the growth rate of output per capita (GDP/cap) but the analysis of this ratio shows that the growth rate of the GDP/cap was also increasing monotonically (Nielsen, 2015a). There was no spurt at all. Furthermore, the analysis of the GDP data (Nielsen, 2016) also shows that there were no spurts (takeoffs) in the growth of the GDP.

When data are closely analysed, they show that what Galor saw as spurts in the growth rates represented just the natural features of monotonically increasing hyperbolic distributions describing the growth of the population, the growth of the GDP and of the growth of the GDP/cap, and of their respective monotonically-increasing growth rates. All these distributions were slow over a long time and fast over a short time. These features are real but they represent nothing mysterious but the natural properties of monotonically-increasing hyperbolic distributions. They create strong illusions of stagnations followed by sudden spurts or takeoffs but when properly analysed they show that there was no stagnation and that the sudden spurts (takeoffs) never happened.

Galor wonders about the relationship between the income per capita (GDP/cap) and the population growth, but the answer to this apparent riddle is simple. When closely analysed, the growth of the population is found to be hyperbolic. The growth of the GDP is also hyperbolic (Nielsen, 2016) and hence, the growth of the GDP/cap is described by the ratio of hyperbolic distributions, which is just a linearly-modulated hyperbolic distribution (Nielsen, 2015a). The mystery is solved.

The only features, which need to be explained, are not the stagnation and sudden spurts (takeoffs) because they did not exist but why the growth of human population and the growth of the GDP were hyperbolic. This issue diverts our attention from phantom problems, which do not need to be solved, and directs it to the problem, which needs to be solved, because if we could explain why the growth of the population and the growth of the GDP were hyperbolic, we could also explain the time dependence of the historical income per capita.

Finally, we shall address a minor issue, which might help to understand at least one discrepancy between the fitted hyperbolic curve and the GDP data (Nielsen, 2016). In that analysis we have found that one point, located at AD 1 was 77% higher than the fitted hyperbolic distribution. In Figure 3 we can see that something similar can be observed for the growth of human population. The size of the population in AD 1 was 71% higher than the size determined by the fitted hyperbolic distribution to the AD data, and the explanation of this discrepancy is simple: there was a maximum in the growth of the population around AD 1 caused by the transition from a fast hyperbolic trajectory during the BC era to a significantly slower hyperbolic trajectory during the AD era. Close similarities between the growth of the GDP and the growth of the population displayed by Maddison's data (Maddison, 2001; 2010) suggest that the 77% difference between the GDP value and the fitted hyperbolic distribution at AD 1 (Nielsen, 2016) might reflect a similar maximum in the growth of the GDP as observed in the growth of the population.

Summary and conclusions

We have analysed the world population data (Manning, 2008; US Census Bureau, 2015) between 10,000 BC and AD 2015. We have found that the growth was hyperbolic during the BC and AD eras.

We have also found that there were just three, relatively, brief demographic transitions during that time: between 500 BC and AD 500, between AD 1200 and 1400 and currently from around 1950. These transitions were of a different kind than usually discussed in academic publications. None of them was a transition from stagnation to a fast growth. None of them represented a sudden takeoff from stagnation to growth, the feature discussed extensively in the Unified Growth Theory (Galor, 2005; 2011).

The first transition was from a fast hyperbolic trajectory to a significantly slower hyperbolic trajectory; the second from a slow hyperbolic trajectory to a slightly faster hyperbolic trajectory; and the current transition from the latest hyperbolic trajectory to a yet unknown trend. The total fraction of time characterising hyperbolic growth was about 89% of the past 12,000 years and the total time taken by transitions was only about 11%. Thus, the analysis shows that if unchecked, population does not increase exponentially as believed by Malthus but hyperbolically. There was also no stagnation in the growth of the world population (Nielsen, 2013a), not only during the AD era but also during the BC era.

Correct understanding of the growth of human population is essential for the correct understanding of economic growth because, as pointed out by Galor (2005; 2011) there is a close relationship between the growth of the population and the growth of income per capita (GDP/cap). We have demonstrated that the growth of the world population was hyperbolic. The growth of the world GDP/cap can be also described using hyperbolic distributions. It is simply a ratio of the hyperbolic distribution describing the growth of the world GDP and the hyperbolic distribution describing the growth of human population (Nielsen, 2015a). Furthermore, it has been already shown that the regional growth of the GDP was hyperbolic (Nielsen, 2016). Similar study could be extended to the growth of regional populations. However, what is already becoming clear is that in order to explain the mechanism of the historical economic growth, expressed either as the GDP or GDP/cap, our attention should be diverted from trying to explain phantom features of stagnation and takeoffs, discussed so extensively in the Unified Growth Theory (Galor, 2005; 2011), the features that did not exist, and that our efforts should be focused on explaining why the economic growth and the growth of human population were hyperbolic.

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7. Population and economic growth in Australia: 8,000 BC – AD 1700 and earlier

Introduction

While data describing economic growth during the AD era are readily available (Maddison, 2001, 2010), similar data for the BC era are hard to find. However, De Long (1998) pointed out that if data for the growth of population are available, they can assist in calculating the Gross Domestic Product (GDP) during the BC era by using the income per capita (GDP/cap) values during the AD era because, income per capita values during the AD era converge quickly to an approximately constant value with the decreasing time (De Long, 1998; Nielsen, 2015). This property, which is nothing more than the mathematical property of dividing hyperbolic distributions (Nielsen, 2015), is mistakenly interpreted as stagnation.

A perfect example of such incorrect interpretation of data is the Unified Growth Theory (Galor, 2005a, 2011). It is a theory based fundamentally on distorted presentation of data and on using impressions created by such distorted presentations. It is an unreliable and misleading theory. The data were used in their distorted way but they were never analysed.

When data are presented in a grossly distorted way (Ashraf, 2009; Galor, 2005a, 2005b, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008), they quickly lead to incorrect conclusions. However, when precisely the same data are analysed, they tell a diametrically different story (Nielsen, 2014, 2015, 2016a, 2016b, 2016c). They show that the Unified Growth Theory and all other similar interpretations of economic and population growth are contradicted by data. In particular, they show that the epoch of Malthusian stagnation did not exist and that there was no escape from the Malthusian trap because there was no trap. Analysis of data shows that the claimed by Galor mysteries of growth did not exist (Nielsen, 2016d, 2016e). It shows that the origin of the claimed mysteries was the distorted presentation of data. Galor created these mysteries by distorting data.

The aim of our discussion presented in this publication is to analyse data for the growth of human population in Australia. As demonstrated earlier (Nielsen, 2016a, 2016f), growth of population and economic growth are closely correlated. They follow nearly identical trajectories. Correct understanding of the growth of population helps also in the correct interpretation of the economic growth.

Data analysis

Rock shelters

Johnson & Brook (2011) analysed the time-dependent distribution of the number of rock-shelter sites in Australia, which they interpreted as representing the growth of ancient human population. The data, as obtained from Brook (2013), are displayed in Figure 1. They are also listed in Table 1. They represent the relative number of rock shelters because they were normalised to 100 at 10,000 years BP. Furthermore, it should be pointed out that in their Figure 4 (Johnson & Brook, 2011) data were shifted by 500 years (Brook, 2013). For instance, the number of rock shelters in 10,000 years BP was *assumed* to represent the number of rock shelters in 9,500 years BP. In our analysis, we shall use the data as supplied by Brook (2013) and as listed in Table 1.

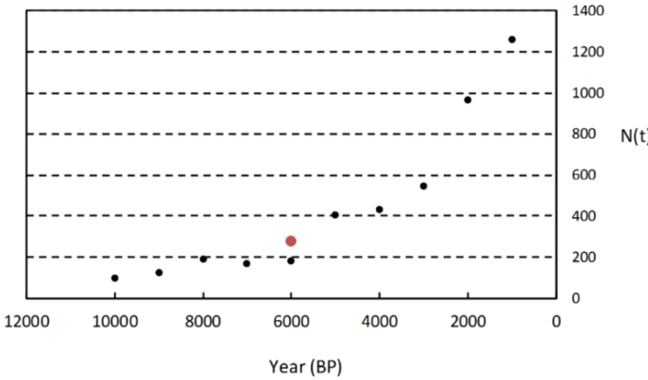

Figure 1. *Data (black dots) for the relative number of rock-shelter sites (Brook, 2013; Johnson & Brook, 2011) representing the growth of human population in Australia. The red dot represents a slightly shifted point at 6,000 years BP illustrating that their claim of the intensification of growth around that time depends entirely on the precise position of this single point belonging to the already inaccurate set of data.*

Table 1. *The relative number of rock shelters,* $N(t)$*, in Australia (Brook, 2013).*

Year (BP)	N(t)	Year (BP)	N(t)	Year (BP)	N(t)
1000	1263	4000	432	7000	168
2000	968	5000	405	8000	189
3000	547	6000	184	9000	126
				10000	100

These data seem to suggest a slow growth until around 6,000 years BP and a faster growth after that year. With their arbitrarily displaced presentation of data by 500 years, the apparent change in the growth pattern could be claimed for 5,000 years BP. Johnson & Brook (2011) concluded that the growth of human population was "slow or negligible before 5000 years ago, and faster since then" (Johnson & Brook, 2011). This observation led them inevitably to the question what might have triggered such a dramatic change in the growth pattern. "Whatever the trigger, our results provide new support for the view, advocated by some Australian archaeologists but contested by others, that something important happened to the human population of Australia during the Holocene, and that the Mid-Holocene in particular was a turning point in Australian prehistory" (Johnson & Brook, 2011).

So now, the vital questions are: Is their conclusion acceptable? Was there or was there not a significant change in the growth pattern of human population in Australia in the distant past? Was there really a turning point in the Australian prehistory? Did something important happen during the Holocene that affected dramatically the growth of population, and consequently also the economic growth?

If there was a change, we have the research field wide open and we can look for answers? However, if the interpretation of data was in some way incorrect and if there was no change, we will have saved a great deal of time, effort and financial resources by not pursuing the suggested line of investigation. We can then divert our efforts into more productive channels.

Before we go any further we should notice that this claim of a sudden intensification of growth around 5,000 years BP (or rather around 6,000 years BP if we plot the data correctly without shifting them by 500 years) depends entirely on *the precise position of just a single point at 6,000 years BP in the already inaccurate set of data*. If this point is shifted only slightly up, as shown in Figure 1, the claim of the intensification is not justified because the data follow then an approximately monotonically increasing distribution. There is no justification for claiming the intensification of growth around 5,000 years BP or around 6,000 years BP. We could terminate our discussion at this stage and conclude that the data give no support to the claim of the intensification of growth. However, data for the growth of human population during the BC era are so rare that, if they become available, it is interesting to analyse them to gain perhaps new information on a related topic.

In order to understand data, it is useful to look at them from a new angle. For instance, semilogarithmic display of data is useful because it identifies easily exponential growth. If data follow approximately a straight line, then the growth is approximately exponential. Data analysed by Johnson and Brook (2011) are presented in Figure 2 using logarithmic scale for the vertical axis.

We can now see clearly that the data follow a *monotonically* increasing trajectory with no sign of any unusual acceleration or intensification. The two phases of growth, fast and slow, did not exist. There was no transition from a slow to a fast growth and there was nothing unusual in the growth pattern around 6,000 years BP [or around 5,000 years BP if we use the arbitrarily shifted data of Johnson & Brook (2011)]. Trying to explain the unusual change in the number of sites around that time or around any other time is irrelevant because there is no convincing evidence that there was a change. On the contrary, in this display, the data follow closely a straight line suggesting exponential growth over the entire range of time.

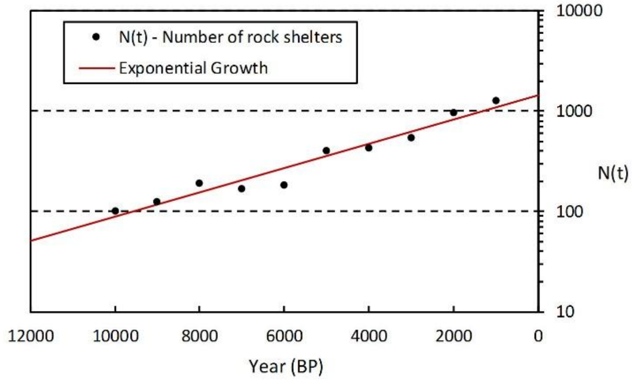

Figure 2. *The number of rock-shelter sites* $N(t)$ *shown in Figure 1 is now plotted using semilogarithmic display. The data follow closely exponential distribution. There is no justification for claiming the intensification of growth around 6,000 years BP*

Exponential distribution is described by the following equation:

$$N(t) = ae^{rt} \tag{1}$$

where, for the distribution presented in Figure 2, $a = 1.114 \times 10^{3}$ and $r = -2.790 \times 10^{-4}$.

The growth rate r is in fact positive but in this equation, it is expressed as negative because the time is expressed in years before present. The number of rock shelters was *increasing* with time.

Another useful way to examine data and to understand their trend is to plot and to analyse their reciprocal values (Nielsen, 2014). This type of display is shown in Figure 3.

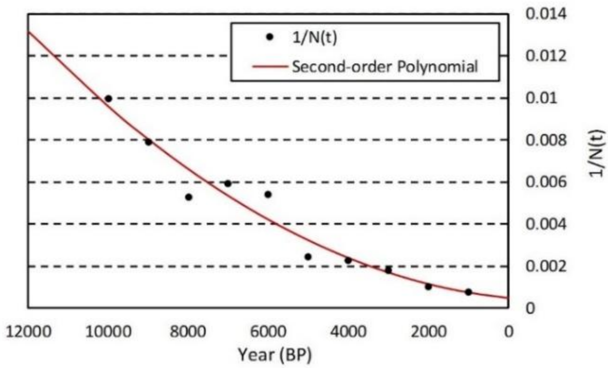

Figure 3. *Reciprocal values,* $1\,/\,N(t)$*, of the number of rock-shelter sites in Australia. There is no sign of any intensification in the number of rock shelters claimed by Johnson & Brook (2011). The best and the simplest fit to the reciprocal values of data is by the second-order polynomial.*

In this representation, an unusual acceleration or intensification in the number of rock shelters would be indicated by a clear *downward* change in the growth pattern. In contrast, data show that the trajectory of the reciprocal values was gradually bending *upwards*. There is no sign of any intensification of growth claimed by Johnson & Brook (2011), not only around 6000 years BP (or around 5000 years BP, depending on how the data are plotted) but also at any time during this section of time. The reciprocal values of data for the number of rock-shelter sites in Australia decrease monotonically with time indicating a monotonic increase in the number of rock shelters. The best and the simplest fit to the reciprocal values of data is by using the second-order polynomial.

We can now combine our analysis of rock shelters in Australia in one figure. Results are presented in Figure 4.

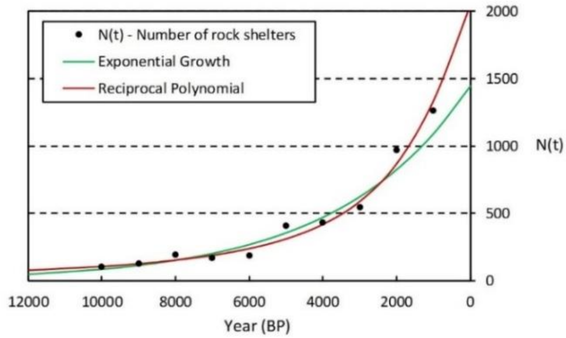

Figure 4. *Mathematical analysis of the number of rock shelters in Australia. The best description of data is by using the reciprocal of the second-order polynomial.*

Initially, the growth of the number of rock shelters can be described well using exponential function or the reciprocal of the second-order polynomial but the reciprocal of the second-order polynomial gives a better overall description of data. This distribution is given by the following equation:

$$N(t) = \left(a_0 + a_1 t + a_2 t^2\right)^{-1} \quad (2)$$

where t is the time in years BP, $N(t)$ is the number of rock-shelter sites, $a_0 = 4.882 \times 10^{-4}$, $a_1 = 1.861 \times 10^{-7}$ and $a_2 = 7.255 \times 10^{-11}$.

So now the puzzling conundrum, acknowledged by some Australian archaeologists (Lourandos, 1997) but contested by others (Hiscock, 2008) has been solved, and the approach is so simple: just a different way of plotting the same set of data and by carrying a simple mathematical analysis of data. Nothing "important happened to the human population in Australia during the Holocene" (Johnson & Brook, 2011) and there was no "turning point in Australian prehistory" (Johnson & Brook, 2011), at least no turning point with respect to the number of rock-shelter sites. There was no trigger and no transition requiring explanation. The number of rock shelters was increasing monotonically over the whole time. The mechanism of the sudden intensification of growth does not have to be explained because there was no intensification.

Growth of population

We can now go a step further and analyse the historical growth of human population in Australia. To this end, we have to translate the number of rock shelters into the size of human population. We shall assume that the size of population was proportional to the number of rock shelters. This approximation works well even if an approximate fixed fraction of the population did not live in rock shelters. For the calibration purpose, we shall use Maddison's data (Maddison, 2010). They overlap the data for the rock shelters at 1000 and 2000 years BP, i.e. at approximately AD 1000 and 1, respectively. The combined data are listed in Table 2 and are also shown in Figure 5 as dots. They extend only to AD 1700 because between AD 1700 and 1800 the steady growth of population was interrupted by the British colonisation. The population in Australia decreased from the estimated 450,000 in AD 1700 to 334,000 in 1820. From around 1840 it started to increase rapidly reaching the first million in 1856 and two million in 1877 (Maddison, 2010). This pattern appears to represent the initial decrease in the aboriginal population followed by the intensified increase in the number of people arriving in Australia.

Table 2. *Growth of human population in Australia, 8,000 BC – AD 1700. The size $S(t)$ is in thousands.*

Year	S(t)	Year	S(t)	Year	S(t)
8000 BC	34	4000 BC	63	AD 1	346
7000 BC	43	3000 BC	139	AD 1000	417
6000 BC	65	2000 BC	149	AD 1500	450
5000 BC	58	1000 BC	188	AD 1600	450
				AD 1700	450

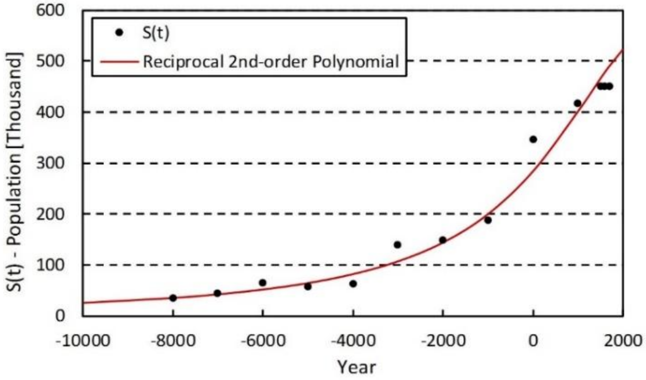

Figure 5. *Growth of human population in Australia, 8,000 BC – AD 1700. The BC years are represented by the negative numbers. The size of population was increasing monotonically. There was no intensification of growth at any time. Growth of population is described remarkably well by the reciprocal of the second-order polynomial [eqn (3)].*

Population in Australia was increasing monotonically between 8,000 BC and AD 1500. There was no intensification of growth at any time. The growth is described well by the reciprocal of the second-order polynomial:

$$S(t) = \left(b_0 + b_1 t + b_2 t^2\right)^{-1} \tag{3}$$

where $S(t)$ is the size of population and t is the time (negative for the BC era). Parameters reproducing the growth of population between 8,000 BC and AD 1500 are: $b_0 = 3.524 \times 10^{-3}$, $b_1 = -1.256 \times 10^{-6}$ and $b_2 = 2.254 \times 10^{-10}$.

This formula reproduces the data so well that it can be used to calculate the size of population at any time between 8,000 BC and AD 1500 or even to extend the estimations to AD 1700 and below 8,000 BC. The calculated values are listed in Tables A1 and A2 in the Appendix. They are close to the empirical values listed in Table 5. There is a certain degree of discrepancy between the predicted values in AD 1600 and 1700. Maddison's data give 450,000 for these two years while the predicted values are 474,000 and 485,000 respectively.

We can also use the determined parameters to calculate the growth rate, which is given by the following formula:

$$R(t) \equiv \frac{1}{S(t)} \frac{dS(t)}{dt} = -\frac{dZ(t)}{dt} S(t), \tag{4}$$

where

$$Z(t) = S^{-1}(t). \tag{5}$$

Explicitly, for the eqn (3), the growth rate

$$R(t) = -(b_1 + 2b_2 t) S(t) = -\frac{b_1 + 2b_2 t}{b_0 + b_1 t + b_2 t^2}. \tag{6}$$

Calculated size of human population in Australia (in thousands) and the corresponding growth rate (in per cent) are shown in Figure 6. The growth rate was increasing steadily but it reached a maximum around AD 1.

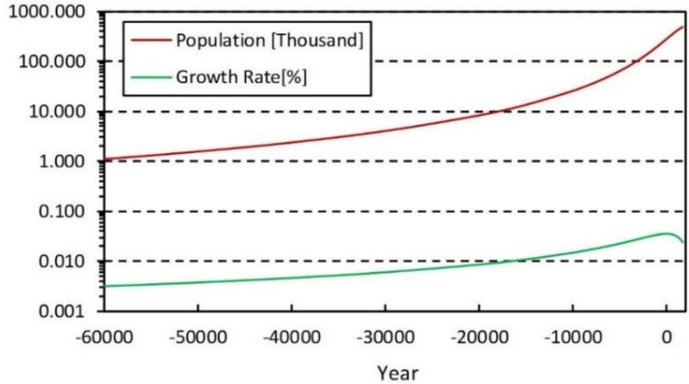

Figure 6. *Calculated size of human population in Australia between 60.000 BC and AD 1700 (in thousands) and the corresponding growth rate (in per cent).*

We should also notice that the parameter $b_2 = 2.254\times10^{-10}$ is small. Mathematical description of the growth of human population in Australia is, therefore, similar to the mathematical description of the historical growth of global and regional populations and to the mathematical description of the historical economic growth (Nielsen, 2016a, 2016f, 2016g). They are all described well using the first-order hyperbolic distributions given by the following simple equation:

$$S(t) = (a - kt)^{-1}. \tag{7}$$

Considering that $b_2 < | b_1 |$,

$$S(t) = \left(b_0 + b_1 t + b_2 t^2\right)^{-1} \approx (b_0 + b_1 t)^{-1} = (a - kt)^{-1}. \tag{8}$$

Distribution, given by the eqn (3) and shown in Figures 5 and 6, is similar to the well-known, and ever-present hyperbolic distribution, given by the eqn (7), which describes so well economic and population growth, global and regional (Nielsen, 2016a, 2016f, 2016g), even down to 10,000 BC for the growth of population. These similarities are shown in Figure 7. The distribution labelled as the *Second-order Hyperbola* (the reciprocal of the second-order polynomial) describes the growth of human population in Australia. It was calculated using the eqn (3) and the empirically determined parameters b_0, b_1 and b_2 listed under this equation. The distribution labelled as the *First-order Hyperbola* (the reciprocal of the first-order polynomial, i.e. the reciprocal of the linear function) was calculated using the eqn (7) and parameters $a = b_0$ and $k = -b_1$. The two distributions differ only by the presence (or absence) of the parameter b_2. For the first-order hyperbolic distribution, $b_2 = 0$. For the second-order hyperbolic distribution $b_2 = 2.254 \times 10^{-10}$. Another essential difference is that, for this set of parameters, the distribution describing the growth of ancient population in Australia does not escape to infinity at a fixed time.

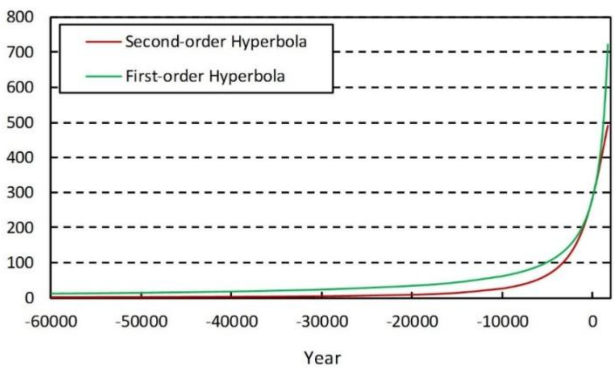

Figure 7. *Characteristic features of the second-order hyperbolic distribution [eqn (3)] describing the growth of population in Australia are similar to the characteristic features of the first-order hyperbolic distribution [eqn (7)]. Parameters used in the calculations are* $b_0 = a = 3.524 \times 10^{-3}$, $b_1 = -k = -1.256 \times 10^{-6}$ and $b_2 = 2.254 \times 10^{-10}$.

Considering the omnipresence of hyperbolic distributions (Nielsen, 2016a, 2016f, 2016g) and that the growth of population in Australia can be so well described using a similar distribution between 8,000 BC and AD 1500 or even 1700, estimation of the size of the population listed in Table A1 was extended tentatively down to 60,000 BC. The widely-accepted date for the arrival of humans in Australia is around 40,000 years ago (Hiscock, 2008) but it could have been also as early as 60,000 years ago (Lourandos, 1997).

There is also a close similarity between the growth of population in Australian and the growth rate calculated using a simpler, first-order hyperbolic distribution. The growth rate for the first-order hyperbolic distribution given by the eqn (7) is

$$R(t) = kS(t) \; . \tag{9}$$

However, considering that for the growth of population in Australia $b_2 < |b_1|$, the corresponding growth rate, given by the eqn (6)

$$R(t) = -(b_1 + 2b_2 t)S(t) \approx -b_1 S(t) = |b_1| \, S(t) \; , \tag{10}$$

because $b_1 < 0$.

Economic growth

According to Maddison (2010), income per capita in Australia between AD 1 and 1700 was constant. Expressed in terms of the 1990 International Geary-Khamis dollars, it was $400. The approximately constant values of income per capita can be easily explained as simply representing the mathematical property of dividing hyperbolic (or hyperbolic-like) distributions (Nielsen, 2015). Using the suggestion of De Long (1998), this property can be used to calculate the past GDP values from the estimated size of population. Economic growth can be assumed to be directly proportional to the size of the population.

The scaling factor for Australia is $400 (1990 International Geary-Khamis dollars). Thus, for instance, the estimated size of Australian population around 40,000 BC is 2,000 and, consequently, the estimated GDP is $800,000. The estimated size of population between 60,000 BC and AD 1700 is listed in Tables A1 and A2. These values can be used to calculate the size of the GDP. The corresponding values after AD 1700 are listed by Maddison (2010).

It is obvious that no-one in Australia, or in any other region for that matter, was calculating the values of the GDP, let alone calculating them in the 1990 International Geary-Khamis dollars in that distant time. The listed values for Australia and for other regions or countries, published by Maddison (2010) for such remote time can serve only as a guide for the *relative* size of the common wealth. Thus, for instance we cannot claim that the value of the GDP in 40,000 BC in Australia was indeed $800,000 but we can estimate that the common wealth in Australia in AD 1700 was about 250 times larger than in 40,000 BC and about 20 times larger than in 10,000 BC. Using the listed values and the values published by Maddison (2010) we can also estimate that the GDP in Australia in AD 2000 was about 5,000 times larger than the common wealth of the aboriginal population around 40,000 BC and about 400 times larger than in 10,000 BC. The estimated growth rate of the GDP below AD 1700, or equivalently the estimated growth rate of the common wealth in Australia is, of course, given by the estimated growth rate of population listed in Tables A1 and A2.

Economic growth in Australia was slow but the growth rate was increasing monotonically until around AD 1, when it started to decrease (see Figure 6). From around that time, the size of the common wealth, as expressed now in terms of the estimated GDP, continued to increase but at the ever-decreasing growth rate. Such a pattern could lead either to a maximum or to the levelling off of the size of the GDP. The use of natural resources by the aboriginal population was exceptionally prudent and parsimonious. Such economic growth could have been sustained practically indefinitely.

Even if the growth rate stopped to decrease from AD 1700 and remained constant, the doubling time for the corresponding exponential growth would have been around 3000 years. The GDP would have increased from $180 million in AD 1700 to only $360 million in around 4700. There was obviously much room for improving the living conditions without the excessively rapid economic growth.

The invasion of Australia changed everything and soon the GDP started to increase rapidly. Rather than doubling in about 3000 years, it doubled in only 135 years soon after AD 1700. By the year 2000, the GDP in Australia increased to $414,058 million. Measured in the constant currency of the 1990 International Geary-Khamis dollars, it was 2300 times larger than in AD 1700. The current growth of the GDP doubles approximately every 22 years. Such a rapid growth is unsustainable.

Summary and conclusions

We have analysed the time dependence of the relative number of rock shelters in Australia. They were assumed by Johnson & Brook (2011) to represent the growth of aboriginal population.

We have found that the growth of population can be best described using the reciprocal of the second-order polynomial. Our analysis shows that within the range of analysable data between 8,000 BC and AD 1700, the generally claimed mythical epoch of the so-called Malthusian stagnation did not exist even in Australia and even in this distant time when early humans must have encountered numerous adverse conditions. Growth of population in Australia was increasing monotonically. It was slow, but definitely not stagnant.

Using the fitted distribution, we have calculated the size of aboriginal population between 8,000 BC and AD 1700. The calculated values are close to the values determined from the study of the number of rock shelters. However, calculations based on the fitted curve allow for filling in the gaps between data.

We have shown that the reciprocal of the second order polynomial, which reproduces the growth of population in Australia, is in the same class as the hyperbolic distributions describing global and regional economic growth and the growth of population (Nielsen, 2016a, 2016f, 2016g). Considering the common presence of hyperbolic distributions and the excellent fit to the data between 8,000 BC and AD 1700, we have tentatively extended our estimates of the size of population in Australia down to 60,000 BC.

It should be remembered, however, that the estimated historical size of Australian population is based on the assumption that it is directly proportional to the relative number of rock shelters. If this assumption is incorrect, then obviously, the estimated size of the population is also incorrect. However, this is the simplest assumption and in science simplest assumptions are usually preferable.

Our analysis solves the puzzle of the so-called Mid-Holocene turning point. According to Johnson & Brook (2011), there was a turning point in the growth of human population in Australia around 5,000 years ago. The growth of population was supposed to have been "slow or negligible before 5000 years BP, and faster since then" (Johnson & Brook 2011). "Whatever the trigger, our results provide new support for the view, advocated by some Australian archaeologists but contested by others, that something important happened to the human population of Australia during the Holocene, and that the Mid-Holocene in particular was a turning point in Australian prehistory" (Johnson & Brook 2011).

This puzzle has now been solved: there was *no* Mid-Holocene turning point in the growth of aboriginal population in Australia. The number of rock shelter sites and the corresponding size of population were increasing *monotonically* between 8,000 BC (approximately 10,000 years BP) and AD 1500 or even 1700. The so-called evidence about the Mid-Holocene turning point is based totally on the position of just a *single* point in the distribution of the already imprecise data (see Figure 1). Relying on just a single point to draw far-reaching conclusions is unacceptable, particularly if, as it is in this case, the data are already inaccurate. Our analysis of data shows that there is nothing remarkable about this single point. It is as close to the calculated distributions as all other points (see Figures 2-5).

With the exception of the recent surge, growth of human population in Australia over the past 10,000 years was remarkably stable and was following closely the distribution described by the reciprocal of the second-order polynomial, which is similar to the commonly observed hyperbolic distributions. Splitting this monotonically increasing growth of population into two distinct segments, as attempted by Johnson & Brook (2011), and trying to explain them by assuming different mechanisms of growth is not only unnecessary but also incorrect. There is nothing to explain about the change in the mechanism of growth because there was no change. However, the data suggest a remarkable and perhaps unexpected feature which could be further investigated. Why was the growth of the ancient human population in Australia so stable, so robust and so resilient to any variable forces over such a long time of around 10,000 years but maybe even over around 60,000 years?

Historical growth of population and historical economic growth in Australia fit well into the generally observed pattern of hyperbolic growth (Nielsen, 2016a, 2016f, 2016g; von Foerster, Mora & Amiot, 1960). Many serious mistakes have been made with the interpretation of such distributions and a good example is the Unified Growth Theory (Galor, 2005a, 2011). These distributions are seen as being made of two distinctly different components, slow and fast. Sometimes a third component is inserted between these two. The perceived slow component is then interpreted as stagnation and the perceived fast component as explosion or takeoff. However, such interpretations are incorrect because hyperbolic distributions increase monotonically. The two distinct components (or stages of growth, or regimes of growth) do not exist. Each hyperbolic distribution or hyperbolic-like distribution, as it is in the case of the growth of ancient population in Australia, has to be interpreted as a whole and the same mechanism has to be applied to the slow and fast growth.

Similar mistake was made by Johnson & Brook (2011) who claimed the intensification of growth around 5,000 years BP. They also divided the monotonically increasing distribution into two stages, slow and fast with an apparent intensification at a certain time. This intensification never happened. They made the same mistake as it is repeatedly made with the interpretation of the historical growth of population and the historical economic growth when the apparent but non-existent intensification is described as takeoff, explosion, sprint or spurt, the features contradicted by the methodical analysis of data.

It is curious that in many publications excellent data are used but they are never analysed (Ashraf, 2009; Galor, 2005a, 2005b, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). It is also curious that the mistake of failing to analyse data is compounded by presenting them in a grossly distorted manner. It is as if data were deliberately manipulated to support erroneous preconceived ideas. Such an approach to research is scientifically unacceptable. It cannot lead to reliable conclusions and in these cases, it did not. All these publications are contradicted by the same data, which in their distorted way were used to promote the erroneous concepts.

Theories such as, the Unified Growth Theory and the Demographic Transition Theory are consistently contradicted by data and by their analyses (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Kapitza, 2006; Kremer, 1993; Lehmeyer, 2004; Livi-Bacci, 1997; Maddison, 2001, 2010; Mauritius, 2015; McEvedy & Jones, 1978; Nielsen, 2014, 2015, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016h, 2016i, 2016j, 2016k; Podlazov, 2002; Shklovskii, 1962, 2002; Statistics Mauritius, 2014; Statistics Sweden, 1999; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994, United Nations, 1973, 1999, 2013; von Hoerner, 1975, von Foerster, Mora & Amiot, 1960; Wrigley & Schofield, 1981). There is no gain in continuing to use these theories. They have to be replaced by new theories which incorporate scientific evidence. In particular, there is no gain in continuing to use the concepts of Malthusian stagnation, Malthusian trap, escape from the Malthusian trap and all other associated concepts because they are contradicted by data and they do not help to explain the mechanism of growth. These concepts are incorrect and misleading. Any attempt to explain the mechanism of the past growth of population or the economic growth should be based on accepting the monotonically increasing hyperbolic distributions

For distributions describing the growth of population and the economic growth, even though the growth was slow it was not stagnant. Even though over a sufficiently long time the growth becomes significantly faster, there is no sudden takeoff or explosion. Hyperbolic distributions can be misleading but their analysis is trivially simple (Nielsen, 2014). Anyone can do it to avoid being misguided by their deceptive features. Hyperbolic distributions are slow over a long time and fast over a short time but they increase monotonically and they cannot be divided into distinctly different stages of growth. The characteristic features of hyperbolic distributions describing the historical economic growth and the historical growth of population should be correctly recognised and accepted in the demographic and economic research.

Appendix

Table A1. *Growth of human population in Australia, 60,000 – 100 BC. The size of population, $S(t)$, is in thousands. The growth rate, $R(t)$, is in per cent (%).*

Year (BC)	S(t) (000)	R(t) (%)	Year (BC)	S(t) (000)	R(t) (%)
60000	1	0.0032	3200	102	0.0274
50000	2	0.0038	3100	104	0.0277
40000	2	0.0047	3000	107	0.0280
30000	4	0.0061	2900	110	0.0283
20000	8	0.0086	2800	114	0.0286
15000	14	0.0110	2700	117	0.0289
10000	26	0.0149	2600	120	0.0292
9500	28	0.0155	2500	124	0.0295
9000	30	0.0161	2400	128	0.0298
8500	33	0.0167	2300	131	0.0301
8000	36	0.0174	2200	136	0.0305
7500	39	0.0181	2100	140	0.0308
7000	43	0.0189	2000	144	0.0311
6500	47	0.0197	1900	149	0.0314
6000	52	0.0207	1800	153	0.0317
5800	54	0.0210	1700	158	0.0320
5600	57	0.0214	1600	164	0.0324
5400	59	0.0219	1500	169	0.0327
5200	62	0.0223	1400	175	0.0330
5000	65	0.0227	1300	181	0.0333
4800	68	0.0232	1200	187	0.0336
4600	71	0.0237	1100	193	0.0338
4400	75	0.0241	1000	200	0.0341
4200	78	0.0247	900	207	0.0344
4000	82	0.0252	800	214	0.0346
3900	84	0.0254	700	222	0.0348
3800	87	0.0257	600	229	0.0350
3700	89	0.0260	500	238	0.0352
3600	91	0.0263	400	246	0.0354
3500	94	0.0265	300	255	0.0355
3400	96	0.0268	200	264	0.0356
3300	99	0.0271	100	274	0.0356

To calculate the GDP, expressed in the 1990 International Geary-Khamis dollars, multiply the size of population, $S(t)$, by $400. The GDP values after AD 1700 are listed by Maddison (2010). Growth rate is the same for the growth of population and for the economic growth.

Table A2. *Growth of human population in Australia, AD 1- 1700.The size of population, $S(t)$, is in thousands. The growth rate, $R(t)$, is in per cent (%).*

Year (AD)	S(t) (000)	R(t) (%)	Year (AD)	S(t) (000)	R(t) (%)
1	284	0.0357	850	382	0.0333
50	289	0.0356	900	388	0.0330
100	294	0.0356	950	395	0.0327
150	299	0.0356	1000	401	0.0323
200	305	0.0355	1050	408	0.0319
250	310	0.0355	1100	414	0.0315
300	316	0.0354	1150	421	0.0310
350	321	0.0353	1200	427	0.0306
400	327	0.0352	1250	434	0.0301
450	333	0.0351	1300	440	0.0295
500	339	0.0349	1350	447	0.0289
550	345	0.0348	1400	453	0.0283
600	351	0.0346	1450	460	0.0277
650	357	0.0344	1500	466	0.0270
700	363	0.0342	1550	472	0.0263
750	369	0.0339	1600	478	0.0256
800	376	0.0336	1650	484	0.0248
			1700	490	0.0240

To calculate the GDP, expressed in the 1990 International Geary-Khamis dollars, multiply the size of population, $S(t)$, by \$400. The GDP values after AD 1700 are listed by Maddison (2010). Growth rate is the same for the growth of population and for the economic growth.

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