Ana Paula Martins

Labor Supply, Demand and Institutional Wage Movements

KSP BOOKS

Labor Supply, Demand and Institutional Wage Movements

Ana Paula Martins
Universidade Católica Portuguesa, Portugal

KSP Books
econsciences.com

Labor Supply, Demand and Institutional Wage Movements

Ana Paula Martins

KSP Books
econsciences.com

ISBN: 978-625-7501-20-0 (e-Book)
KSP Books 2021

Labor Supply, Demand and Institutional Wage Movements

Author: **Ana Paula Martins**
Universidade Católica Portuguesa, Portugal

econsciences.com

Preface

Chapter 1. The analysis discusses the labor market equilibrium under union oligopoly, where unions represent homogeneous workers and use employment strategies. The following points are addressed: 1. The labor market outcomes in the presence of; a. uncooperative behavior among unions; b. uncooperative environment with a leading union; c. collusive (coordinated unions) behavior among unions; d. globally efficient bargaining, are confronted. A specific example with a Stone-Geary utility function and linear demand is forwarded. 2. Supply dynamics may push up employment and, therefore, the number of unions. In equilibrium, some bounds exist to the number of unions the market can support, which are investigated in the example. Five supply dynamics are considered: a. reservation wage restriction; b. a standard labor supply constraint; c. number of unions equal demand; d. individualistic unions; e. existence of a minimum (employed) membership requirement. The equilibrium number of unions for the Cournot-Nash, Stackelberg and efficient bargaining structures is derived for the case where unions exhibit Stone-Geary preferences and labor demand is linear.

Chapter 2. In this chapter we present and confront the expected outcome of a raise in earnings taxes on the regional or sectoral allocation of labor force and employment. The basic frameworks are the benchmark dualistic scenarios. A single-input analysis of an homogeneous product economy is provided once extensions were designed to highlight the role of mobility barriers and how they interact with local wage-setting rules to determine regional allocation rather than trade issues or factor substitution. We report the main effects on equilibrium local after-tax wages, supply, employment and aggregate welfare surplus of a unilateral as well as a simultaneous unit tax increase of the (a) basic two-sector model in six different scenarios: free market; partial (one-sector) coverage with perfect intersector mobility; partial (one-sector) coverage with imperfect mobility (Harris-Todaro); multiple (two-sector) coverage with imperfect mobility (Bhagwati-Hamada); partial (one-sector) coverage with affiliation restrictions in the covered sector; partial (one-sector) coverage with limited employment generation ability in the traditional uncovered sector. Needless to say, the results would apply to any other production factor, one or other scenario being more appropriate for inference of the consequences of differential taxation systems.

Chapter 3. This chapter analyzes the labor market outcome when there are two unions in the industry, representing homogeneous workers (hence, unions use employment strategies), in the presence of union employment (quantity) constraints. Three strategic environments are considered: Nash-Cournot duopoly, Stackelberg duopoly and efficient cooperation between the two unions. Employment constraints -ceilings and floors- originate kinks and/or discontinuities in the reaction functions, leading to corner solutions and special features of the labor market equilibrium. Two types of constraints are discussed. One is insufficient employed membership (ceiling) for the interior solution. Then it may be optimal for a Stackelberg leader to push the other to the bound. The other case considered is the legal requirement of a minimum number of employed members that the union must have to be constituted. Entry-deterrence strategies of the leading union may then emerge.

Dr. Ana Paula Martins
Lisboa, Portugal
21 September 2021

Contents

1 Union oligopoly and entry in the presence of homogeneous labor

Introduction

In the two last decades, union strength and importance in wage determination seem to have decreased substantially; it is the main contribution of this article to offer a methodological understanding of the dynamics that may be behind such process. Hence, this research derives and compares the features of the labor market equilibrium in which several unions intervene under different strategic environments and follows to the "economics of union collapse" by allowing the number of unions in the economy to increase.

The multiple union setup has been modeled and its implications studied by authors as Oswald (1979)[1], Gylfason

[1] Citing Rosen (1970) as the first author to recognize strategic interdependency among unions.

& Lindbeck (1984a) and (1984b) and others[2]. These studies usually assume heterogeneous labor with imperfect substitutability between workers and consider price, i.e., wage competition.

We are going to focus, instead, on homogeneous labor and consider that unions, as Hart´s (1982) syndicates, use employment strategies[3]. The use of these strategies is also a realistic assumption: legal restrictions to unemployment practices and laws restricting temporary labor contracts can be seen as part of the bargaining outcome; also, employment seems to be the concern of some strikes against firm bankruptcy or maintenance of partial contracts. This context has already been analyzed by the authors - Martins & Coimbra (1997)[4] - for the duopoly case. The present research is oriented towards the continuation of that research, and the extension of the study to the determination of the equilibrium number of unions the market may support.

We therefore start by considering the equilibrium conditions of four environments which differ with respect to the union and employer competitive (cooperative) behavior: Cournot-Nash strategies; Stackelberg equilibrium; efficient bargaining among the unions; efficient bargaining among unions and employers (which correspond to contract curve agreements in the one union case). This is presented in section II.

Section III is designed to derive more specific conclusions with respect to the labor market outcome through the use of

[2] Also Davidson (1988), Dixon (1988), Dowrick (1989), Jun (1989) and Dobson (1994), for example, where the effect of the existence of oligopoly in the product market is investigated.

[3] Notice that even if unions effectively consider price, i.e., wage, strategies, there will be equivalent quantity-employment strategies that will reproduce the former. See Martins & Coimbra (1997) for a justification. Also, Martins (1998) for the appraisal of dual reaction functions in the presence of heterogeneous labor.

[4] The reader is referred to the literature review there cited.

Stone-Geary union preferences and, when necessary, a simple linear demand schedule. Symmetric solutions - i.e., in which unions' utility functions are alike - are also analyzed, allowing us to derive conclusions about the relation between the equilibrium wage and employment and the number of unions in the market.

In the presence of a fixed number of unions, labor demand determines equilibrium employment and wage. If there is unemployment, and a sort of closed-shop scenario in which employment - at least for that demand - can only be achieved through unionization, it is reasonable to conceive that unemployed (unionized and/or non-unionized) workers will form their own unions and push employment up - and/or wages down [5]. The supply pressure - that can be seen as due to "outsiders" [6] (in Lindbeck & Snower's (1988) sense) or unsatisfied "insiders" reaction when legally allowed - will probably cause the number of unions to raise, at least till a certain point. This is the subject of section IV.

Different unions may be formed because labor force participants have different preferences over the employment-wage mix - therefore, membership assignment is a result of differentiated preferences of the labor force. But we do not pursue here a link to the membership dynamics literature. We are interested in union formation rather than union affiliation and argue that, as for firms in the product market model, the labor market may only support a fixed number of unions.

Notice that union entry is really a movement towards collapse of union effectiveness, even in a closed-shop

[5] As (positive) profit opportunities would attract new firms in the product market.

[6] We do not pursue a model of insider-outsider justification - as Solow (1985), for an example. Instead, we deal with an environment where "outsiders existence" is only justified by the closed shop agreement and/or some other legal requirement.

scenario. If we consider industry-wide bargaining, union formation in most Western Countries is not usually restricted by labor legislation, apart from some representativeness requirements. Also, wage setting is commonly generalized to the whole industry or economy, which approaches the closed shop philosophy. Therefore, our study, even if theoretical, mirrors empirical realities.

The ways in which supply (or other) restricts union formation may be varied. We put forward five scenarios. Some of these are solutions with standard usage in the macroeconomics literature - the existence of a reservation wage, below which workers do not accept jobs, and a standard linear labor supply schedule. Others are suggested by the union scenario: aggregate employment goes up till the point where everybody who is unionized is (fully) employed - i.e., equilibrium number of unions equals demand; there is a fixed (exogenous) number of individuals in the economy which will form that many unions (in equilibrium they will/may not be fully employed). Finally, we consider another realistic situation: there is a legal requirement on the minimum number of employed members a union must exhibit to be considered representative in labor negotiations.

The solutions for Nash-Cournot unions, Stackelberg equilibrium and efficient bargaining unions are compared for symmetric unions with Stone-Geary utility functions and for a linear labor demand schedule.

The exposition ends with a summary of the main conclusions in section V.

Union oligopoly and other solutions[7]

Assume that there are n unions in the economy. The unions maximize the general utility function $U^i(L_i,W)$, increasing in the arguments and quasi-concave, for which U^i_L / U^i_W - the marginal rate of substitution between employment and wage - decrease with L_i and increases with W. Employment contracts are under closed-shop agreements, i.e., the firm(s) can only hire unionized workers. Demand is of the form:

$$\sum_{i=1}^{n} L_i = L(W) \qquad (1)$$

or its inverse:

$$W = W(\sum_{i=1}^{n} L_i) \qquad (2)$$

being negatively sloped, coming from maximization (in $L = \sum_{i=1}^{n} L_i$) of the (aggregate) profit function $\Pi(L,W) = P F(\sum_{i=1}^{n} L_i) - W \sum_{i=1}^{n} L_i$. Therefore, (2) establishes the value

[7] This section's results are a generalization of those presented in Martins & Coimbra (1997) for the two unions case, being, thus, presented in a sketchy manner.

of the marginal product of labor, equal for all types of workers[8].

Cournot oligopoly

Each union maximizes

$$\max_{L_i, W} U^i(L_i, W) \qquad (3)$$

s.t.: $\sum_{i=1}^{n} L_i = L(W)$ or $W = W(\sum_{i=1}^{n} L_i) = P F_L(\sum_{i=1}^{n} L_i)$, or

$$\max_{L_i} U^i[L_i, W(\sum_{i=1}^{n} L_i)] \qquad (4)$$

The optimal solution will obey

$$U^i_L / U^i_W = - W_L \quad \text{or} \quad L_i = R^i(L_1, \ldots, L_{i-1}, L_{i+1}, \ldots, L_n) = R^i(\sum_{j\neq i} S\ L_j) \qquad (5)$$

$$i=1,2,\ldots,n$$

where $R^i(\sum_{j\neq i} S\ L_j)$ denotes union i's reaction function, and labor demand[9].

[8] Nevertheless, most of the results below would also apply if this function represented the marginal revenue product of labor and if firms did not behave competitively in the product market.

[9] Existence of equilibrium is guaranteed by concavity of each union i's utility function with respect to L_i, and uniqueness is satisfied if $\Sigma_{i=1}^{n}$

Union 1 is Stackelberg leader

The optimal solution will obey labor demand and

$$U^i_L + U^i_W W_L = 0 \quad \text{or} \quad L_i = R^i(L_1, ..., L_{i-1}, L_{i+1}, ..., L_n)$$

$i=2,...,n$ (6)

One can solve the system of n-1 equations in n unknowns in such a way that:

$$L_i = R'^i(L_1) \quad , \quad i=2, ..., n \tag{7}$$

Then, the leader, say 1, solves:

$$\underset{L_1, L_2,..., L_n}{\text{Max}} \quad U^1[L_1, W(\sum_{i=1}^{n} L_i)] \tag{8}$$

$$\text{s.t.:} \quad U^i_L + U^i_W W_L = 0 \quad \text{or} \quad L_i = R^2(L_1), \ i = 2, 3, ..., n$$

Equilibrium is defined by (7), demand and:

$$U^1_L / U^1_W = - W_L (1 + \sum_{j=2}^{n} dR'^j / dL_1) \tag{9}$$

$$= (1 + \sum_{j=2}^{n} dR'^j / dL_1) \ U^i_L / U^i_W \ , \ i = 2, 3, ..., n$$

$dR^i/dL_{-i} / (1+dR^i/ dL_{-i}) \leq 0$, - where $dR^i/dL_{-i} = dR^i/d(\Sigma\ S_{j\neq i}\ L_j)$ - which will hold if $-1 \leq dR^i / dL_{-i} \leq 0$, i=1,2,...,n. This ensures that optimal L_i falls as L rises. See Friedman (1983), p. 30-33.

Efficient cooperation

Assume the unions cooperate with each other and we can extend the Nash-maximand approach to more than the 2 unions scenario. Then, unions maximize:

$$\underset{L_1, L_2, \ldots, L_n}{\text{Max}} \quad \prod_{i=1}^{n} \{U^i[L_i, W(\sum_{j=1}^{n} L_j)] - \bar{U}^i\}^{\delta_i} \tag{10}$$

δ_i is related to the strength of union i within the coalition, as justified by Svejnar (1986), extending the Nash-Zeuthen-Harsanyi solution. With $\delta_n = 1$, δ_i can be associated in "fair gambles" with M_i/M_n, where M_i denotes number of members of union i. If we consider that $\sum_{i=1}^{n} \delta_i = 1$, then we can link $\delta_i = M_i / \sum_{j=1}^{n} M_j$ [10].

Eventually, (10) could represent the utility function of a unique union with workers with different preferences over the wage-employment mix, having n types of workers, with M_i workers of type i, i =1, 2,..., n. With perfect substitution between workers, ultimately the wage paid by the firm(s) must be the same for all workers.

F.O.C yield:

[10] See Martins & Coimbra (1997) for a justification of the relation between δ_i and number of members of union i.

$$\delta_i U^i_L / [U^i(L_i, W) - \bar{U}^i] = - W_L \sum_{j=1}^{n} \delta_j U^j_W / [U^j(L_j, W) - \bar{U}^j] \quad (11)$$

$$i = 1, 2, ..., n$$

Therefore, as the right-hand-side is the same for any i:

$$\delta_i U^i_L / [U^i(L_i, W) - \bar{U}^i] = \delta_j U^j_L / [U^j(L_j, W) - \bar{U}^j] \quad (12)$$

This can be seen as a distribution (across unions) equation.

In this case, the equilibrium will obey labor demand and [11]:

$$\sum_{i=1}^{n} U^i_W / U^i_L = - L_W = - 1 / W_L \quad (13)$$

As we have seen [12], this case reproduces the monopoly union behavior.

[11] Efficiency conditions, in Edgeworth tradition, would also come from the solution of the problem

$$\max_{L_1, L_2, ..., L_n, W} U^1(L_1, W)$$

$$\text{s.t.:} \quad U^j(L_j, W) \geq \bar{U}^j \ , \quad j = 2, 3, ..., n$$

$$W = W(\textstyle\sum_{i=1}^{n} L_i)$$

[12] Martins & Coimbra (1997).

Fully efficient bargaining

$$\underset{L_1,\dots,L_n,W}{\text{Max}} \prod_{i=1}^{n} [U^i(L_i, W) - \bar{U}^i]^{\delta_i} \; [\Pi[(\sum_{i=1}^{n} L_i), W] - \bar{P}] \qquad (14)$$

δ_i represents the strength of union i relative to the employer side, hence, δ_i / δ_j represents the strength of union i relative to union j. A bargaining with equal strength between unions (together) and employers will require:

$$\sum_{i=1}^{n} \delta_i = 1 \qquad (15)$$

F.O.C. will yield (12) and also [13]:

$$\sum_{i=1}^{n} U^i_W / U^i_L = \Pi_W / \Pi_L = - \sum_{i=1}^{n} L_i / [P\,F_L - W] \qquad (16)$$

Final remarks

The comparison of the four forms for a union duopoly can be found in Martins & Coimbra (1997) [14]; without a

[13] These properties would also arise from the solution of the problem

$$\underset{L_1, L_2, \dots, L_n, W}{\text{Max}} \; U^1(L_1, W)$$

$$\text{s.t.:} \quad U^j(L_j, W) \geq \bar{U}^j \;, \quad j = 2, 3, \dots, n$$

$$\Pi(\sum_{i=1}^{n} L_i, W) \geq \bar{P}$$

[14] See Table 1 of that work for a summary of the marginal rate of substitution conditions.

particular form for the union utility function and labor demand not much can be add. Again, the main important features of the solutions are:

- the equality of the sum of the marginal rates of substitution between wage and labor in the coalitions cases, i.e., with efficient cooperation among the unions - as opposed to equality of each of such rates when unions compete as Cournot - to the slope of labor demand - or Π_W / Π_L -, thus suggesting a higher wage relative to employment when unions collude.

- a Stackelberg leader would pick a point where its marginal rate of substitution $U^1{}_L$ / $U^1{}_W$ is higher than the followers', suggesting a higher L_1 than if she were a follower - and higher than the followers' employment if the utility functions of all the unions are similar.

- efficient cooperation with the employer side leads to an equilibrium where wages are higher than the marginal product of labor - as it occurs in the traditional contract curve solution when there is only one union.

An analytical example

Assume that unions maximize the special case of the Stone-Geary utility function:

$$U^i(L_i,W) = W^{\theta_i} L_i^{(1-\theta_i)} \tag{17}$$

$\gamma_i = (1 - \theta_i) / \theta_i$ represents union i's relative (to wage) preference for employment. Whenever necessary, a linear demand schedule is going to be considered:

$$W = a - b\,(\sum_{i=1}^{n} L_i) \tag{18}$$

Oligopoly equilibrium

From F.O.C., we can derive:

$$[\theta_i / (1 - \theta_i)]\, s_i = \eta \tag{19}$$

where denotes union i´s employment share, i.e., $s_i = L_i / L$, and:

$$s_i = [(1 - \theta_i) / \theta_i] / [\sum_{j=1}^{n} (1 - \theta_j) / \theta_j] \tag{20}$$

$$\eta = 1 / [\sum_{i=1}^{n} (1 - \theta_i) / \theta_i] \tag{21}$$

Therefore:

Proposition 1: With Stone-Geary utility functions as above in a oligopoly union

1. the equilibrium employment share of each union is independent of the form of the production function (i.e., of labor demand)

2. the share of employment of each union will be equal to the weight of γ_i in total sum of γ_i's.

3. the market will lead to an equilibrium where the elasticity of demand is equal to the inverse of the total sum of the θ_i's.

The reaction functions are of the type:

$$L_i = a(1-\theta_i)/b - (1-\theta_i)(L-L_i) \quad (22)$$

Solving for L_i as a function of L:

$$L_i = (a/b)(1-\theta_i)/\theta_i - (1-\theta_i)L/\theta_i \quad (23)$$

Summing over i, we get:

$$L = \sum_{i=1}^{n} L_i = a/b \sum_{i=1}^{n} (1-\theta_i)/\theta_i - L \sum_{i=1}^{n} (1-\theta_i)/\theta_i \quad (24)$$

Solving for L, we get:

$$L = (a/b)\left[\sum_{i=1}^{n} (1-\theta_i)/\theta_i\right] / \left[1 + \sum_{i=1}^{n} (1-\theta_i)/\theta_i\right] \quad (25)$$

Using the labor demand schedule, we conclude that:

$$W = a / \left[1 + \sum_{i=1}^{n} (1-\theta_i)/\theta_i\right] \quad (26)$$

Replacing (25) in (23),

$$L_i = (a/b)\left\{[(1-\theta_i)/\theta_i] / \left[1 + \sum_{j=1}^{n} (1-\theta_j)/\theta_j\right]\right\} \quad (27)$$

We can therefore conclude that:

Proposition 2: In an oligopoly of unions with Stone-Geary preferences and linear labor demand:

1. $\gamma = \sum_{i=1}^{n} (1 - \theta_i) / \theta_i$ is a measure of aggregate union preference for labor relative to wage.

2. Equilibrium aggregate employment is positively related to γ and negatively related to b, the slope of the demand.

3. The wage is inversely related to γ.

4. Each union's employment is positively related to $\gamma_i = (1 - \theta_i) / \theta_i$, the measure of the union preference for employment relative to wage.

Let us consider a symmetric equilibrium.

If all unions optimize the same utility function, then $\theta_i = \theta$, i=1,2,...,n. In this case, we will have that:

$$L = (a / b)\ 1 / \{ 1 + \theta / [n (1 - \theta)] \} \tag{28}$$

$$W = a / [1 + n (1 - \theta) / \theta] \tag{29}$$

$$L_i = (a / b) \{ 1 / [n + \theta / (1 - \theta)] \} \tag{30}$$

We therefore see that:

Proposition 3: In an oligopoly of unions with Stone-Geary preferences and linear labor demand:

1. W and L_i decrease with n; therefore, also W L_i, each union's wage bill will decrease with n.

2. L increases with n. But one can show that WL will decrease with n:

3. As $n \rightarrow \infty$, $W \rightarrow 0$ and $L \rightarrow a / b$ ($WL \rightarrow 0$).

4. n = 1 solves the monopoly union problem.

Stackelberg Equilibrium

We want to analyze the situation where a union, say 1, acts as leader and the others are followers. (6) and (8) hold. (6) yields:

$$L_i = a(1-\theta_i)/b - (1-\theta_i)\left(L_1 + \sum_{j=2}^{n} L_j - L_i\right), \quad i = 2,3,\ldots n \tag{31}$$

Solving for L_i,

$$L_i = \left[a/b - \left(L_1 + \sum_{j=2}^{n} L_j\right)\right](1-\theta_i)/\theta_i, \quad i = 2,3,\ldots n \tag{32}$$

Summing over i = 2,..,n

$$\sum_{i=2}^{n} L_i = \left[a/b - \left(L_1 + \sum_{j=2}^{n} L_j\right)\right]\sum_{i=2}^{n}(1-\theta_i)/\theta_i \tag{33}$$

Solving in order to $\sum_{i=2}^{n} L_i = \sum_{j=2}^{n} L_j$,

$$\sum_{i=2}^{n} L_i = (a/b - L_1)\sum_{i=2}^{n}(1-\theta_i)/\theta_i \Big/ \left[1 + \sum_{i=2}^{n}(1-\theta_i)/\theta_i\right] \tag{34}$$

Replacing in (32) we get

$$L_i = [(1-\theta_i)/\theta_i]\,(a/b - L_1)\Big/\left[1 + \sum_{j=2}^{n}(1-\theta_j)/\theta_j\right], \quad i = 2,3,\ldots n \tag{35}$$

Maximizing union 1's utility function with respect to the system yields:

$$L_1 \theta_1 b / [1 + \sum_{i=2}^{n} (1 - \theta_i) / \theta_i] = (1 - \theta_1) W = \tag{36}$$

$$= (1 - \theta_1)\{a - b L_1 - b \sum_{i=2}^{n} (1 - \theta_i) / \theta_i (a/b - L_1) / [1 + \sum_{i=2}^{n} (1 - \theta_i) / \theta_i\}$$

Solving for L_1 :

$$L_1 = (1 - \theta_1) a/b \tag{37}$$

$$W = a \theta_1 / \{[1 + \sum_{i=2}^{n} (1 - \theta_i) / \theta_i]\} \tag{38}$$

For the followers:

$$L_i = (a/b) [(1 - \theta_i)/ \theta_i] \theta_1 / \{[1 + \sum_{j=2}^{n} (1 - \theta_j) / \theta_j]\} \tag{39}$$

$$L = (a/b) \{1 - \theta_1 / [1 + \sum_{i=2}^{n} (1 - \theta_i) / \theta_i]\} \tag{40}$$

Comparing with (37)-(40) to (24)-(27) for given n, we conclude

Proposition 4: 1. W is now lower - L will be higher - than it was with no leader.

2. Each follower's quantity is lower than in the case of Cournot oligopoly.

Assume that $\theta_i = \theta$, i = 1,2,...,n. Then:

$$L_1 = (1 - \theta)\ a/b \tag{41}$$

$$L_i = (a/b)\ (1 - \theta) / [1+ (n - 1)\ (1 - \theta) / \theta\ ] \tag{42}$$

$$L = (a/b)\ \{1 - \theta / [1 + (n -1)\ (1- \theta) / \theta\ ]\ \} \tag{43}$$

and

$$W = a\ \theta / [1 + (n -1)\ (1-\theta) / \theta\ ] \tag{44}$$

Efficient Bargaining

Assume, as usual, that $\bar{U}^i = 0$, i = 1, 2,...,n. In this setting, (12) yields:

$$\delta_i\ (1 - \theta_i) / L_i = \delta_j\ (1 - \theta_j) / L_j \tag{45}$$

Therefore:

$$s_i = \delta_i\ (1 - \theta_i) / \sum_{j=1}^{n} \delta_j\ (1 - \theta_j) \tag{46}$$

(13) yields:

$$\sum_{i=1}^{n} [\theta_i / (1 - \theta_i)]\ s_i = \eta \tag{47}$$

and:

$$\eta = \sum_{i=1}^{n} \delta_i\ \theta_i\ / \sum_{i=1}^{n} \delta_i\ (1 - \theta_i) = \tag{48}$$

$$= [\sum_{i=1}^{n} \delta_i \theta_i / \sum_{i=1}^{n} \delta_i] / \{[1 - [\sum_{i=1}^{n} \delta_i \theta_i / \sum_{i=1}^{n} \delta_i] =$$

$$= \overline{\theta} / (1 - \overline{\theta})$$

where

$$\overline{\theta} = \sum_{i=1}^{n} \delta_i \theta_i / \sum_{i=1}^{n} \delta_i \qquad (49)$$

i.e., $\overline{\theta}$ is the weighted average of the θ_i's, the weights being the strength parameters in the coalition.

Proposition 5: With Stone-Geary utility functions and efficient bargaining between unions (i.e., coordination of union bargaining):

1. the equilibrium employment share of each union is independent of the form of the production function
2. the employment share of each union will be higher, the lower θ_i, the union's preference for wage, and the higher θ_i, the union's strength parameter.
3. the employment share of union i will be higher than in the Cournot game iff

$$\delta_i \theta_i > \sum_{j=1}^{n} \delta_j (1-\theta_j) \; / \; \sum_{j=1}^{n} (1-\theta_j) / \theta_j$$

4. the elasticity of demand will, in equilibrium, be higher than in the point where the Cournot game end.
5. The elasticity of aggregate demand will be equal to the monopoly union solution for a union the θ of which equals the weighted (by the δ_i's) average of the θ_i's.

If demand is linear, one can show that:

$$L_i = (a/b)\,\delta_i\,(1 - \theta_i) \,/ \sum_{j=1}^{n} \delta_j \quad , \quad i = 1,2,..,n \tag{50}$$

$$L = (a/b) \sum_{i=1}^{n} [(1-\theta_i)\,\delta_i] \,/ \sum_{i=1}^{n} \delta_i = (a/b)\,(1 - \bar{\theta}) \tag{51}$$

and

$$W = a \sum_{i=1}^{n} \theta_i\,\delta_i \,/ \sum_{i=1}^{n} \delta_i = a\,\bar{\theta} \tag{52}$$

We can see that the aggregate labor market outcome is analogous to the monopoly union solution and does not depend on the number of unions in the industry (or in the economy) - suggesting that a higher number of unions will just have, at least on average, lower employment.

One can show that:

Proposition 6: In a equilibrium, with Stone-Geary union preferences and a linear demand schedule, with unions behaving cooperatively among themselves:

1. W is higher and aggregate employment is lower than in Cournot oligopoly.

2. Aggregate employment and wage is invariant to the number of unions in the economy, i.e., no matter how many unions there are in the economy, they will share the same aggregate employment.

3. In a symmetric equilibrium, i.e., $\theta_i = \bar{q} = \theta$ and $\delta_i = \delta$ for all i, W will be higher and employment will be lower than in the Stackelberg equilibrium.

4. In a symmetric equilibrium, each union's employment will be lower than in the Cournot case. Each follower will

have a lower employment than in the Stackelberg case if $\theta > 0{,}5$.

Globally Efficient Bargaining

Then we can show that (45) and (46) hold and distribution of employment is the same as with only efficient bargaining among unions. Contract curve agreements will be such that:

$$W = (a - b\,L)\,\bar{\theta} \,/\, (2\,\bar{\theta} - 1) \tag{53}$$

Notice that this expression is equivalent to the contract curve with only one union. The number of unions does not affect (directly) the aggregate contract curve relation.

If $\bar{\theta} > 0{,}5$, for a positive (meaningful) wage, the marginal product of labor will be positive. If $\bar{\theta} < 0{,}5$ - in which case unions' preferences for employment (relative to wage) are very high -, for a positive wage, we must have in equilibrium aggregate employment till a point where the marginal product of labor is negative.

Therefore:

Proposition 7: In fully efficient bargaining with union Stone-Geary preferences and a linear demand curve, considering positive wage results, in equilibrium solutions:

1. If $\bar{\theta} > 0{,}5$, the marginal product of labor will be positive.

If $\bar{\theta} < 0{,}5$, aggregate employment will be pushed till a point where the marginal product of labor is negative. This result is independent of unions relative strength to the employers side.

2. For given aggregate employment, contract curve agreements will imply a larger wage level the larger is the average union preferences for wage, $\bar{\theta}$.

3. The contract curve relation does not depend on the number of unions.

The effective solution depends on the δ_i's and turned out to be of difficult comparison with the previous cases. We concluded that, if unions maximize the wage bill, i.e., $\theta = 0{,}5$ and, for $\delta_i = \delta$ for all i:

$$L = (a / b) \tag{54}$$
$$L_i = (a / b)(1 / n) \tag{55}$$

The wage will (also) depend on the relative strength of the unions and employers.

Union entry

Introduction

Assume there is a fixed number of unions in the labor market. They set employment and membership is exogenous. If labor supply at the equilibrium wage is much higher than employment, unemployed workers may press wages down in order to get a job - that is, supply reacts. As we consider closed-shop agreements, and recalling some of the oligopoly symmetric solutions, aggregate employment increases with n, i.e., the number of unions. Then, it is reasonable to suppose that unemployed workers (unionized or not...) collude, form a (or other) new union(s) and push the wage down till they get employment.

We can derive the equilibrium number of unions, n*, in several ways:

1. Reservation Wage Restriction.

We can assume there is a reservation wage W_r below which unions collapse. With entrance, wage decreases till this point is reached. Alternatively, W_r may represent a minimum wage floor, arising from (exogenous) general legislation.

2. Labor Supply Constraint.

Another form is to assume labor supply reacts directly, pressing down wages till unions are virtually irrelevant in wage and aggregate employment determination, only maintained through the legal closed shop requirements. There is a labor supply schedule (or a membership demand function), which will be reached.

Notice that this outcome guarantees that full-employment will be achieved.

3. Number of Unions Equals Demand.

Each employed worker behaves as a union, i.e., $L_i = 1$ (at least for non-leaders). Eventually, each union's utility function can be seen as representing each individual's preferences over wage W and probability of employment, L_i [15]; then, $n^* = L$ (for Cournot-Nash equilibrium; for Stackelberg, $n^* - 1 = L - L_1$) establishes the n* that will guarantee full-employment (but only) of unionized workers.

In terms of insider-outsider theory, this corresponds to the solution of the maximum number of insiders the (closed) system supports - which depends on demand, and union preferences.

4. Individualistic Unions.

If each individual (not necessarily employed all the time...) behaves as an union and there are $\bar{L}$ individuals in the market, then a bound for n is $\bar{L}$.

This could be seen as the limit solution, when the union utility function represents, as before in 3., the worker's

[15] A similar interpretation of the household maximand can be found in Oswald (1979).

preferences over the probability of (un)employment (and, indirectly, leisure) - wage mix . The final outcome would originate a Natural Rate of Unemployment, given by $(\bar{L} - \sum_{i=1}^{n} L_i) / \bar{L}$, as long as $L_i < 1$; this could be seen as underemployment solution, as $L_i > 1$ would correspond to overemployment cases.

This scenario is only meaningful in the Stackelberg case if we admit that it applies exclusively to the workers not employed in union 1, i.e., $n^* - 1 = \bar{L} - L_1$. It would be as if union 1 members would be full-employed insiders.

5. Minimum Employed Members Requirement.

A realistic scenario is that there is an exogenous - legally required - minimum number of employed members, M, that each union must exhibit to be constituted or considered representative for access to the closed-shop, i.e., $L_i \geq M$. Union entrance will occur till this bound is reached.

We can appreciate the dynamics of union formation in the Cournot environment. Or in the Stackelberg case. In the latter, at first glance, the scenario could be seen as describing the insider-outsider environment, with members of union 1 corresponding to insiders. However, insiders have the same wage as outsiders; only, the leader has now fixed employment - independent of n; but, as in the product market, the leader ends up by sustaining the reduction in quantity to support the wage.

Finally, note that, from (51)-(52) we conclude that with union collusion, the aggregate outcome is invariant to n. Nevertheless, conclusions can be drawn for some of the five cases.

To derive the equilibrium n, n*, we admit symmetric unions, i.e., $\theta_i = \theta$ and $\delta_i = \delta$ for all i. We ignore problems (indivisibilities) arising from treating n as a rational number. We also derive the final labor market outcome - W, L and L_i - for the endogenous n*.

Nash-Cournot equilibrium

The derivation of results starts from equations (28)-(30).

A. Reservation Wage Restriction.

Then, if all unions are similar, using (29), we will have union "entrance" till:

$$W = W_r = a / [1 + n(1 - \theta)/\theta] \tag{56}$$

That is, the maximum number of unions the market will support will be:

$$n^* = (a / W_r - 1)\ \theta / (1 - \theta) \tag{57}$$

n* is positively related to the unions relative preference for wage and negatively related to the reservation or minimum wage. Replacing (57) in (28) and (30):

$$L = (a - W_r) / b \tag{58}$$

$$L_i = W_r\ [(1 - \theta) / \theta] / b \tag{59}$$

B. Labor Supply Constraint

We will have (28) and (29) and, say, a linear labor supply schedule (or a membership demand function):

$$W = c + d\,L \tag{60}$$

Solving for n*:

$$n^* = [b(a-c)/(bc+da)]\ \theta/(1-\theta) \qquad (61)$$

The equilibrium wage and employment in this case will be the same as with no unions - i.e., supply equals demand.

$$L = (a-c)/(b+d) \qquad (62)$$
$$W = (bc+da)/(b+d) = b\ (L/n^*)\ \theta/(1-\theta) \qquad (63)$$
$$L_i = [(1-\theta)/\theta]\ (bc+da)/[b\ (b+d)] \qquad (64)$$

We expect that $n^* < L$, i.e., each union will employ more than one individual, $L_i > 1$.

C. Number of Unions Equals Demand

If each employed worker behaves as the union, then, $n^* = L$ and

$$L_i = 1 \qquad (65)$$

Using in (30), we can solve for:

$$n^* = L^* = a/b - \theta/(1-\theta) \qquad (66)$$

Notice that now n^* is negatively related with the unions relative preference for wage and negatively related to the inverse of the labor demand slope, b. Eventually, each union's utility function can be seen as representing each individual's preferences over wage W and probability of employment, L_i; then, condition (66) establishes the n^* that will guarantee full-employment of unionized workers.

Also

$$W = b\ \theta/(1-\theta) \qquad (67)$$

W will be the same as in (63) for $n^* = L$.

D. Individualistic Unions

If each individual (not necessarily employed all the time...) behaves as an union and there are $\bar{L}$ individuals in the market, then a bound for n is $\bar{L}$ and (28)-(30) with n replaced by $\bar{L}$ will be the bounds for W, L and L_i - with L_i representing the equilibrium probability of employment of each individual. We will therefore have:

$$n^* = \bar{L} \tag{68}$$

$$L = (a / b) \; 1 / \{ 1 + \theta / [\bar{L} (1 - \theta)] \} \tag{69}$$

$$W = a / [1 + \bar{L} (1 - \theta) / \theta] \tag{70}$$

$$L_i = (a / b) \{ 1 / [\bar{L} + \theta / (1 - \theta)] \} \tag{71}$$

$$L_i \leq 1, \text{ iff}$$

$$\bar{L} \geq (a/b) - \theta / (1 - \theta) \tag{72}$$

E. Minimum Union Employed Members Requirement

There is an exogenous minimum number of employed members, M, that each union must exhibit to be constituted. Then, entrance will occur till the point where:

$$L_i = M \tag{73}$$

Using (30) we conclude that

$$n^* = (a/b)(1/M) - \theta / (1 - \theta) = [a(1 - \theta) - b M \theta] / [b M (1 - \theta)] \quad (74)$$

It varies negatively with M and ◎, the union preference for wage relative to employment.

$$L = (a/b) - M\theta / (1 - \theta) = [a(1 - \theta) - b M \theta] / [b(1 - \theta)] \quad (75)$$

and

$$W = b M \theta / (1 - \theta) \quad (76)$$

Proposition 8: 1. The equilibrium number of unions increases with the unions' relative preference for wage in the reservation wage and the labor supply constraint cases. It decreases when number of unions equals demand and when there is employed membership requirement.

2. A reservation wage level higher than the final equilibrium wage in all other cases will imply a smaller number of unions in equilibrium.

3. In general, we expect individualistic unions to originate a higher number of unions than any other case (provided only that $L_i > 1$ for the labor supply constraint).

4. We expect that the minimum employed members requirement implies the lowest equilibrium number of unions (provided only that the requirement is higher than the equilibrium employment per union of the other cases).

One can give an intuition for the result 1. of Proposition 7. In the first two cases, we have, in fact, a labor supply constraint: the reservation wage hypothesis corresponds to a infinitely elastic labor supply. Therefore, supply and demand determine total employment - and wage. In these circumstances, the number of unions can be seen as determined from wage equation (29), showing that the wage varies negatively with n and positively with θ - as expected;

therefore the positive relation between n and θ follows immediately.

In the case where the number of unions is demand determined, the stronger are the insiders preferences for employment - weaker for wage - the larger will be aggregate employment - for given n. As in this case n is simultaneously determined by demand, we arrive at the conclusion that n* should increase with (1 - θ) - increase with θ.

The explanation for the minimum employed members requirement case can invoke the relation (30): in a Cournot equilibrium, each union's employment varies inversely with n and θ. Then, if L_i is fixed, the relation between n and θ is straightforward.

Stackelberg Equilibrium

To derive the equilibrium number of unions, we consider equations (41)-(44). Employment of the leader, L_1, does not depend on n and is always fixed and equal to:

$$L_1 = (1 - \theta)\, a/b \qquad (77)$$

Considering:

A. Reservation Wage Restriction

Then:

$$n^* = \theta / (1 - \theta) \left[a\, \theta / W_r - (2\theta - 1) / \theta \right] \qquad (78)$$

We can show that the new n* will be smaller than for Cournot oligopoly iff $W_r / a < \theta$; from (44) we can see that this always occurs. Total employment is the same as in the Cournot case and given by (58).

B. Labor Supply Constraint

As we have seen, total employment and wage is fixed outside the control of the union - given by

$$L = (a - c) / (b + d) \tag{79}$$

$$W = (bc + da) / (b + d) \tag{80}$$

Then,

$$n^* = \theta / (1 - \theta) [a \theta (b+d) / (bc + ad) - (2 \theta - 1) / \theta] \tag{81}$$

C. Number of Unions Equals Demand

I.e., for outside unions $L_i = 1$.

$$n^* = a \theta / b - (2 \theta - 1) / (1 - \theta) \tag{82}$$

If $a/b > 1 / (1 - \theta)$, i.e., for the leader, $L_1 > 1$, which we implicitly assume, then the new n* will be smaller than in the Cournot case.

In this case:

$$L = (1 - \theta) a/b + (n^* - 1) = a/b - \theta / (1 - \theta) \tag{83}$$

Therefore, total employment is maintained relative to Cournot oligopoly. Only a redistribution favoring the leader will take place.

D. Individualistic Unions

We admit the restriction applies to workers not employed in union 1 - the insiders which would have full-employment. We will therefore have:

$$n^* - 1 = \bar{L} - L_1 = \bar{L} - (1 - \theta) a/b \tag{84}$$

Then:

$$n^* = \bar{L} - L_1 + 1 = \bar{L} + 1 - (1 - \theta)\ a/b \qquad (85)$$

n* will be smaller than in the Nash-Cournot case. Also:

$$L = (a / b)\ (1 - \theta / \{1 + [\bar{L} - (1 - \theta)\ a/b]\ (1 - \theta) / \theta]\}\ ) \qquad (86)$$

$$W = a\,\theta\ / \{1 + [\bar{L} - (1 - \theta)\ a/b]\ (1 - \theta) / \theta]\} \qquad (87)$$

$$L_i = (a / b)\ \theta / \{\theta / (1 - \theta) + [\bar{L} - (1 - \theta)\ a/b]\} \qquad (88)$$

As in the Nash-Cournot case, $L_i \leq 1$ iff

$$\bar{L} \geq a/b - \theta / (1 - \theta) \qquad (89)$$

E. Minimum Union Employed Members Requirement

M is going to restrict the n-1 followers:

$$L_i = M \quad , \quad i = 2, 3, \ldots, n \qquad (90)$$

We can see then that:

$$n^* = \theta\ a/(b\ M) - (2\,\theta - 1) / (1 - \theta) \qquad (91)$$

From (43),

$$L = (a/b) - M\,\theta / (1 - \theta) \qquad (92)$$

From (44),

$$W = b\ M\,\theta / (1 - \theta) \qquad (93)$$

As in previous cases, n* is smaller than in the analogous Cournot solution iff $\theta > W/a = (b\ M\ /a)\ \theta\ /\ (1 - \theta)$, which - from (44) - must always occur. However, the aggregate solution (L, W) is the same as in the corresponding Cournot case.

Proposition 9: 1. Stackelberg equilibrium:

1. in general, will originate a smaller number of unions than the corresponding Nash-Cournot case.

2. will lead to the same aggregate employment and equilibrium wage than the corresponding Nash-Cournot case.

Efficient Bargaining

We consider equations (50)-(52) and assume the symmetric case where $\theta_i = \theta$ and $\theta_i = \theta$ for all i. It will always be the case that:

$$L_i = (a/b)\ (1 - \theta) / n = L / n \quad , \quad i = 1,2,\ldots,n \tag{94}$$

$$L = (a/b)\ (1 - \theta) \tag{95}$$

and

$$W = a\ \theta \tag{96}$$

Therefore, total employment and wage are fixed and do not depend on n.

B. Labor Supply Constraint

In this case, denote the supply wage, W^S and effective supply L^S at the efficient bargaining result, which is independent of n:

$$W^S = c + d\ (a/b)\ (1 - \theta) \tag{97}$$

$$L^S = (a\ \theta - c) / d \tag{98}$$

If $W = a\theta > c + d(a/b)(1 - \theta)$, there will be involuntary unemployment at rate:

$$(L^S - L) / L^S = \{a[b\theta - d(1 - \theta)] - cb\} / [b(a\theta - c)] \qquad (99)$$

However, if $W = a\theta < c + d(a/b)(1 - \theta)$, there won't be enough supply.

Alternatively, we can admit that in any case number of people

$$n^* = L^S = (a\theta - c) / d \qquad (100)$$

and number of available jobs are given by (95). Each "supply unit" will work:

$$L / L^S = (a/b)(1 - \theta) / [(a\theta - c) / d] \qquad (101)$$

This may be smaller than 1 (corresponding to the involuntary unemployment case), or larger than 1.

Notice that this case is not directly comparable to the Cournot or Stackelberg cases, once there is now unemployment.

C. Number of Unions Equals Demand

If each employed worker behaves as the union, then, $n^* = L$ and

$$L_i = 1 \qquad (102)$$

Then:

$$n^* = (a/b)(1 - \theta) \qquad (103)$$

D. Individualistic Unions

We will therefore have, as long as $L_i \leq 1$:

$$n^* = \bar{L} \tag{104}$$

and

$$L_i = (a/b)\ (1 - \theta) / \bar{L} \quad , \qquad i = 1,2,\dots,n \tag{105}$$

$L_i \leq 1$, iff:

$$(a/b)\ (1 - \theta) \leq \bar{L} \tag{106}$$

E. Minimum Union Employed Members Requirement

Then:

$$L_i = M \tag{107}$$

Using (94) we conclude that

$$n^* = (a/b)\ (1 - \theta) / M \tag{108}$$

Proposition 10: Efficient bargaining:

1. will imply that the equilibrium number of unions will vary with unions preference for wage as in the Cournot case, except in the minimum membership requirement case (in which they now vary inversely).

2. will originate a smaller aggregate employment and higher wage than any Nash-Cournot or Stackelberg cases (except for the labor supply constraint).

3. will imply a smaller number of unions than the corresponding Nash-Cournot case in the minimum

employed members requirement iff $\theta < 0{,}5$ (a larger number iff $\theta > 0{,}5$); a smaller number in number of unions equal demand.

Summary and conclusions

This chapter gathers some notes and enlargements to the standard collective bargaining problem in which unions maximize utility; we oriented the analysis to model union formation.

The research started by confronting different scenarios of union and firm(s) strategic behavior. The results - some are summarized in Table 1 - are an extension of the duopoly case, previously studied. A specific example using Stone-Geary union utility functions is derived - see Table 2 for some of the main labor market outcomes. We conclude that efficient cooperation among unions can be seen as a union composite behaving as a monopoly union; cooperation between unions and employers reproduce contract curve agreements for the one union solution.

The symmetric equilibrium (with an exogenous number of unions) allows us now to infer that with non-cooperative (among themselves) union behavior - as expected -, the equilibrium wage will decrease with the number of unions and employment will move in the opposite direction. Yet, with efficient bargaining among unions, aggregate outcomes are invariant to the number of unions in the industry.

The equilibrium (endogenous) number of unions - which will presumably rise while there is unemployment - is studied for five cases: there is a reservation or minimum wage restriction; a standard labor supply constraint closes the model; number of unions equals demand; individualistic unions, the number of which is exogenous and equal to the number of individuals in the economy; there is a legal minimum employed members requirement. Minimum employed membership requirements, with free union entry, partly restore union effectiveness in wage lifting. The results are summarized in Table 3. Interestingly, it is not always the case - as one would expect - that number of unions increases with the unions' preferences for wage relative to employment. Number of unions will be larger (or at least equal) in Cournot-Nash than in Stackelberg environments; both will have the same aggregate outcome, i.e., wage and total employment.

Efficient bargaining will support, in general, a smaller (or not larger) number of unions than Cournot, except when minimum employed membership requirements are imposed (With a labor supply constraint, results are not comparable). Aggregate employment will be smaller in efficient bargaining - and wage higher - than in Cournot or Stackelberg cases in the final equilibrium with endogenous number of firms.

Table 1. *Marginal Rate of Substitution Conditions*

	Equation	Efficiency Locus
A. Cournot-Nash	(5)	$U^i_L / U^i_W = - W_L$, $i = 1,2, \ldots, n$
B. Stackelberg	(9)	$U^1_L / U^1_W = (1 + \sum_{j=2}^{n} dR^j / dL_1)\, U^i_L / U^i_W =$ $= - (1 + \sum_{j=2}^{n} dR^j / dL_1)\, W_L$, $i = 2, 3, \ldots, n$
C. Efficient Union Cooperation	(13)	$\sum_{i=1}^{n} U^i_W / U^i_L = - L_W$
D. Globally Efficient Cooperation	(16)	$\sum_{i=1}^{n} U^i_W / U^i_L = \Pi_W / \Pi_L =$ $= - (\sum_{i=1}^{n} L_i) / [P\, F_L - W]$

Table 2. *Equilibrium Outcomes for Stone-Geary Union Utility Functions*

Variable	Cournot Oligopoly	Stackelberg Oligopoly
L_1	(27) $(a/b)(1/\gamma_1)/[1+\sum_{j=1}^{n}(1/\gamma_j)]$	(37) $a(1-\theta_1)/b$
L_i	(27) $(a/b)(1/\gamma_i)/[1+\sum_{j=1}^{n}(1/\gamma_j)]$	(39) $(a/b)[(1-\theta_i)/\theta_i]\,\theta_1 / \{[1+\sum_{i=2}^{n}(1-\theta_i)/\theta_i]\}$
L	(25) $(a/b)[\sum_{i=1}^{n}(1-\theta_i)/\theta_i] / [1+\sum_{i=1}^{n}(1-\theta_i)/\theta_i]$	(40) $(a/b)\{1-\theta_1/[1+\sum_{i=2}^{n}(1-\theta_i)/\theta_i]\}$
W	(26) $a/[1+\sum_{i=1}^{n}(1-\theta_i)/\theta_i]$	(38) $a\theta_1/\{[1+\sum_{i=2}^{n}(1-\theta_i)/\theta_i]\}$
Variable	**Efficient Union Cooperation**	**Globally Efficient Cooperation (for $\theta_i = 0.5$; $\delta_i = \delta$)**
L_i	(50) $(a/b)\,\delta_i(1-\theta_i)/\sum_{j=1}^{n}\delta_j$	(55) $(1/n)\ a/b$
L	(51) $(a/b)\sum_{i=1}^{n}[(1-\theta_i)\delta_i] / \sum_{i=1}^{n}\delta_i = (a/b)(1-\bar{\theta})$	(54) a/b
W	(52) $a\sum_{i=1}^{n}\theta_i\delta_i/\sum_{i=1}^{n}\delta_i = a\bar{\theta}$	(Dependent on δ)

Table 3. *Equilibrium Number of Unions*

	Cournot-Nash	Stackelberg	Efficient Bargaining
A. Reservation Wage Restriction	(57) $(a/W_r - 1)\ \theta/(1-\theta)$	(78) $\theta/(1-\theta)\ [a\theta/W_r -$ $- (2\theta - 1)/\theta]$	-
B. Labor Supply Constraint	(61) $[b(a-c)/$ $/(bc+da)]\ \theta/(1-\theta)$	(81) $\theta/(1-\theta)\ [a\theta$ $(b+d)/(bc+ad)\ -$ $- (2\theta - 1)/\theta]$	(100) $(a\theta - c)/d$
C. Number of Unions Equals Demand	(66) $a/b - \theta/(1-\theta)$	(82) $a\theta/b -$ $- (2\theta - 1)/(1-\theta)$	(103) $(a/b)(1-\theta)$
D. Individualistic Unions	(68) $\bar{L}$	(85) $\bar{L} - L_1 + 1 =$ $= \bar{L} + 1 - (1-\theta)\ a/b$	(104) $\bar{L}$
D. Minimum Union Member Requirement	(74) $a/(bM) - \theta/(1-\theta)$	(91) $\theta\ a/(bM) -$ $- (2\theta - 1)/(1-\theta)$	(108) $(a/b)(1-\theta)\ M$

References

Davidson, C. (1988). Multiunit bargaining in oligopolistic industries. *Journal of Labor Economics*, 6(3), 397-422. doi. 10.1086/298189

Dixon, H. (1988). Unions, oligopoly and the natural range of employment. *Economic Journal*, 98(393), 1127-1147. doi. 10.2307/2233723

Dobson, P.W. (1994). Multifirm unions and the incentive to adopt pattern bargaining in oligopoly. *European Economic Review*, 38(1), 87-100. doi. 10.1016/0014-2921(94)90007-8

Dowrick, S. (1989). Union-oligopoly bargaining. *Economic Journal*, 99(398), 1123-1142. doi. 10.2307/2234092

Friedman, J. (1983) *Oligopoly Theory.* Cambridge: Cambridge University.

Gylfason, T., & Lindbeck, A. (1984a). Competing wage claims, cost inflation and capacity utilization. *European Economic Review,* 24(1), 1-21. doi. 10.1016/0014-2921(84)90010-2

Gylfason, T., & Lindbeck, A. (1984b). Union rivalry and wages: An oligopolistic approach. *Economica*, 51, 129-139. doi.

Hart, O. (1982). A model of imperfect competition with Keynesian features. *Quarterly Journal of Economics,* 97(202), 109-138. doi. 10.2307/2554205

Jun, B.H. (1989). Non-cooperative bargaining and union formation. *Review of Economic Studies*, 56(1), 59-76. doi. 10.2307/2297749

Lindbeck, A., & Snower, D.J. (1990). *The Insider-Outsider Theory of Employment and Unemployment*. Macmillan.

Martins, A.P. (1998). Union duopoly with heterogeneous labor. *Working Paper*, No.88.98. Lisboa, Universidade Católica Portuguesa.

Martins, A.P., & Coimbra, R. (1997). Union duopoly with homogeneous labor. *Working Paper*, No.84.97. Lisboa, Universidade Católica Portuguesa.

Oswald, A.J. (1979). Wage determination in an economy with many trade unions. *Oxford Economic Papers,* 31(3), 369-85. doi. 10.1093/oxfordjournals.oep.a041454

Rosen, S. (1970). Unionism and the occupational wage structure in the United States. *International Economic Review,* 11(2), 269-286. doi. 10.2307/2525669

Solow, R.M. (1985). Insiders and outsiders in wage determination. *Scandinavian Journal of Economics*, 87(2), 411-428. doi. 10.2307/3439843

Svejnar, J. (1986). Bargaining power, fear of disagreement, and wage settlements: Theory and evidence from U.S. industry. *Econometrica*, 54(5), 1055-1078. doi. 10.2307/1912322

2 Taxation and mobility in dualistic models– (and) Some neglected issues of fiscal federalism

Introduction

The aim of this research is to contrast the expected long-run impact of taxation on labor force flows under alternative scenarios, highlighting the interplay of three environment features: institutional wage setting, barriers to mobility, and differential tax treatment across regions.

The subject is of interest to international factor mobility analysis, where income tax treatment of foreign residents and their consequences has deserved considerable debate (Bhagwati, 1982). In this area, skill distribution biases seem to be of major concern (See Bhagwati & Hamada, 1982, for example), as well as potential evasion – either legally, through emigration, or illegally through fraud.

At the national affairs level, fiscal federalism faces the same mechanisms on the revenue generation side. However, going over the literature (Oates, 1999) provides a recent survey. Musgrave & Musgrave (1976), Atkinson & Stiglitz

(1980), Tresch (1981) all contain fiscal federalism sections), it more frequently aims at redistribution issues, or the optimal level of local expenditures (in the tradition of the Tiebout (1956) insights). Optimal local taxation has, of course, been addressed (see Mieszkowski & Zodrow (1989) for an appraisal) – sometimes oriented towards exploring the existence and/or consequences of decentralization (after Gordon's (1983) good starting point), but generally assuming an underlying competitive labor or factor market.

Our goal was to expose the potential distortionary effects of differentiated labor income taxation schemes under different assumptions of mobility across regions, jurisdictions, or productive sectors, and to recognize that if compounded with non-competitive restrictions – unionisation, for example – it may imply quite unexpected efficient or second-best recommendations.

We focus on the impact of taxation working through potential labor flows – of taxpayers "voting with their feet" – and employment allocation. Redistribution of the revenue levied on the local population is superimposed – but, as a lump-sum, not affecting workers response, potentially arising from a general public good provided to both regions' consumers in an uniform manner. Under such assumptions, one could predict that a local earnings factor tax would have similar effects to an institutional wage floor; and such conclusion is traditionally cited in the general equilibrium two factor-two sector analysis textbook example (Layard & Walters (1978), section 3-5 and exercise Q3-10; Bosworth, Dawkins & Stromback (1996), section 10.3.2. Also, Johnson & Mieszkowski (1970)). As we will see, under different mobility assumptions, that is not at all the case. Complementarily, we compute the expected total welfare changes implied by the different fiscal environments - local assessments in these matters are difficult to disentangle, once (as noted) after a fiscal change individuals' affiliation also change.

The basic structures chosen to replicate the effects of taxation were simple dualistic models in the tradition of Harris (1969) and Harris-Todaro (1970) rural-urban migration analysis (a good survey of theoretical literature can be found in Bhattacharya (1993)). The principles behind its workings became widespread in the study of labor market regional as also sector – occupation, profession – allocation (see McNabb & Ryan (1990) for a survey of segmented labor markets. See also Saint-Paul (1996) for applications of the theory with microfoundations for several dualistic structures.) and under minimum or other wage legislation or restrictions (see, for example, Mincer (1976), McDonald & Solow (1985) and Fields (1989). Also Brown, Gilroy & Kohen (1982)). We follow the cases contrasted in Martins (1996), inspecting the consequences of introducing a local unit tax [1] on employment in each of the scenarios.

Total – national, worldwide according to context we may wish to simulate - labor force supply is assumed perfectly inelastic. The hypothesis was thought convenient once it neutralizes the welfare tax – the "deadweight" - loss under a uniform tax system.

Also, neither workers – that chose location, or sector affiliation, maximizing the expected after-tax wage - nor unions – setting net wages - possess any "tax illusion".

After notation is briefly settled in section I, we depart from the benchmark case - free market with perfect mobility across regions or sectors -, outlined in section II. In section III, partial coverage with perfect mobility - i.e., people not employed in the primary sector can immediately get a job in the secondary sector and wait there for an opportunity to switch, and thus, there is (again) no unemployment

[1] Proportional earnings taxes would generate similar qualitative outcomes.

generation - is introduced. In section IV, a version of the Harris-Todaro model - with imperfect mobility and institutionally fixed wage in one of the sectors - is inspected. In section V, the Bhagwati-Hamada economy – with two covered sectors - is forwarded. Section VI deals with frameworks where there are size restrictions: in the primary sector size - the counterpart of the H.-T. model; and in the employment generation capacity of the secondary sector - the "dual" case of the B.-H. model. The exposition ends with a brief summary in section VII.

Notation

There are two sectors - or two regions - and a fixed exogenous labor supply, $\bar{L}$. This total labor supply decides whether to locate in region (or affiliate to sector) 1 or 2. Denote by $\bar{L}_i$ local/industry supply in region/sector i. Then:

$$\bar{L}_1 + \bar{L}_2 = \bar{L} \tag{1}$$

An inelastic supply in single-sector models is known to generate no deadweight loss. The assumption is thus useful to assess tax implications with respect to the reallocation of resources in dualistic frameworks – potentially implying welfare losses that would not arise with uniformity and total mobility.

In sector i, the aggregate demand function is given by:

$$L_i = L^i(W_i) \quad , \quad i = 1, 2 \tag{2}$$

A non-positive slope – that is, $L^i(W_i)' = dL^i(W_i)/dW_i \leq 0$ – is always assumed. Denote the corresponding inverse demand function by:

$$W_i = W^i(L_i) \quad , \quad i = 1, 2 \tag{3}$$

There are no cross effects, i.e., $dL^i/dW_j = 0$ for $i \neq j$. The wage elasticity of demand of sector i at a particular point of labor demand will be denoted by

$$\varepsilon^i = L^i(W_i)' \, W_i \, / \, L^i(W_i) = W^i(L_i) \, / \, [W^i(L_i)' \, L_i]. \tag{4}$$

The background technology and preferences in the economy are anything but complex: an homogeneous good is produced and consumed in both regions. Identical workers as "land-owners" consume directly what they produce or receive as income: there is no (reason to) trade, nor (need for) money. Also, there is no inter-regional (sectoral) imbalance in terms of affiliation preferences: all the individuals value is income and the regions do not differentiate otherwise by any intrinsic characteristics.

We will assume further through sections II to V a subset of the following:

1. individuals are risk neutral and maximize expected (after-tax) income.

2. there is no fiscal (tax) illusion and institutional wages are set in net terms.

3.a. there is perfect mobility across sectors alternatively:

3.b. job rotation is only accomplished locally or within the industry.

4.a. wage in sector 1 is determined by market conditions; alternatively:

4.b. wage in sector 1 is institutionally determined.

5.a. wage in sector 2 is market determined; alternatively:

5.b. wage in sector 2 is institutionally determined.

Assumptions 1 and 2 insure that comparative statics may be adequately performed for a model – equilibrium conditions - set in reference to net or after-tax wages and are always superimposed. For simplicity, assume a unit tax T_i per unit of employment is levied on sector i. Then, labor demands can be expressed as:

$$L_i = L^i(W_i + T_i) \quad ; \quad \text{and} \quad W_i = W^i(L_i) - T_i \quad , \; i = 1, 2 \qquad (5)$$

where W_i is the after-tax wage. One might argue that the assumptions ignore the possibility of higher taxes being re-collected in the form of higher local public expenditures; then, a raise in the local tax could have similar effects to that of a local minimum wage. We adopt those assumptions to measure the effect of a distortionary tax under "non-distortionary" expenditures: say, the whole economy's fiscal revenue is evenly distributed by the total labor force. (Or they can chose in which region to spend vacations - reside - and consume the local public good). They will be appropriate if we are inspecting industry-specific rather than regional labor force flows.

In another angle, a tax on one region's employment – specially in an uncovered region – is expected to have the same effect as a negative relative local public expenditure disadvantage – say, the region requires higher or more expensive infrastructure to maintain its equal appeal to the other. Or a negative effect due to relative location intrinsic cost of living – a requirement of more expensive heating/housing costs due to climatic reasons – or appeal; on a sector's employment, of a relative worker distaste for industry affiliation – say, a compensating differential is required for a riskier profession. These imbalances would generate – and provide a rationale for - a persistent or long-run equilibrium differential of expected wages – even if not of expected individual welfare - across sectors or regions in favor of one of the areas in the absence of government intervention. The models' mechanics would not be much altered if we added an exogenous differential "a" to the expected wages of the favored region in the equilibrium conditions below. One should keep in mind that such a differential would not create the need for intervention under a complete mobility assumption – maximization of total welfare (including the externality "consumption" by residents of the favored region) would be guaranteed by a competitive market equilibrium dynamics.

Assumptions 3 to 5 characterize the mobility environment. Different combinations of alternatives a and b generate backgrounds of benchmark dualistic models that we shall stage. For instance, 3.a. insures that there will be no unemployment in the economy – provided that the wage is market determined in at least one of the sectors.

In the presence of one sector only and inelastic supply, a tax has no aggregate welfare consequences – of course, under the utilitarian social-welfare criteria underlying consumer surplus(or economic rent)-based analysis -, provided the net wage is free. With institutionally set wages – and because supply is inelastic – the welfare – total surplus - change from an increase in the tax rate can be approximated in monetary terms by, in space (L, W), the area below the (inverse) labor demand schedule between the before and after-tax employment levels. If $L_i(T_1, T_2)$ is the equilibrium employment in sector i when tax levels are, respectively, T_1 and T_2 in sectors 1 and 2, the welfare surplus of the two-sector/region economy is:

$$\int_0^{L_i(T_1,T_2)} W^i(L_i)\, dL_i \; + \int_0^{L_j(T_1,T_2)} W^j(L_j)\, dL_j \qquad (6)$$

Then, the effect on sector i of a unitary change in region j's tax rate can be approximated by:

Δ_i = Welfare change in sector i per unit of ∂T_j = $W^i(L_i)$ $\partial L_i(T_1, T_2)/\partial T_j$

The total effect in the two areas of the tax levied on sector j would naturally add to:

Total Welfare Change per unit of ∂T_j =

$$= W^i(L_i)\, \partial L_i(T_1, T_2)/\partial T_j \; + \; W^j(L_j)\, \partial L_j(T_1, T_2)/\partial T_j \qquad (7)$$

Such expressions simplify in each of the simulated environments.

We may want to contrast total revenue generation ability of a unitary tax raise, using the fact that revenue levied on region j is:

$$RF_j = L^j(W_j + T_j)\; T_j \qquad (8)$$

and thus:

$$\partial RF_j/\partial T_j = L^j(W_j + T_j) \; + \; T_j\, \partial L_j/\partial T_j \qquad (9)$$

For simplicity, we will be interested in measuring effects for T_j around 0. Then, the welfare change implied by an unitary increase in fiscal revenue obtained from region j will be higher the lower is $L^j(W_j + T_j)$. That is:

Welfare change (total, or in sector i) per unit of ∂RF_j =

$$[\text{Welfare change (total, or in sector i) per unit of } \partial T_j] \,/\, [L^j(W_j + T_j) + T_j\, \partial L_j/\partial T_j\,] \quad (10)$$

To obtain one extra (a given…) monetary revenue unit, the government would extract it by raising the tax rate T_j for which that measure - for the total economy - is lower. Under well-behaved second-order conditions and existence of interior solutions, an efficient scheme (T_i, T_j) – guaranteeing a minimum $RF_i + RF_j \leq R$ - would equalize the (total welfare change per unit of ΔRF_i) to the (total welfare change per unit of ΔRF_j).

Competitive labor markets under perfect mobility

Assume 3.a, 4.a and 5.a. of the previous section. Then:

$$\bar{L}_i = L_i = L^i(W_i + T_i)\ ,\quad i = 1,2 \quad (11)$$

In the present scenario, people will move from one to the other sector's employment till equalization of net wages. That is, after-tax wage W will adjust till W* that solves:

$$L^1(W^* + T_1) + L^2(W^* + T_2) = \bar{L} \quad \text{or}$$

$$W^* = W^i(L_i) - T_i = W^j(L_j) - T_j = W^j(\bar{L} - L_i) - T_j \quad (12)$$

The increase of the tax rate has mainly effects on the regional allocation of labor and on the net wage bill. The net wage decreases:

$$\partial W^*/\partial T_i = - L^i(W^* + T_i)' / [L^1(W^* + T_1)' + L^2(W^* + T_2)'] < 0 \quad (13)$$

$$\partial L_i/\partial T_i = -$$

$$\partial L_j/\partial T_i = L^1(W^*+T_1)'L^2(W^*+T_2)'/[L^1(W^*+T_1)'+L^2(W^*+T_2)'] =$$

$$= 1/[1/L^1(W^*+T_1)' + 1/L^2(W^*+T_2)'] < 0 \quad (14)$$

Given perfect mobility and inelastic supply, there will be no unemployment but there will be some "deadweight loss" as long as there is some reallocation of labor resources relative to the free market solution:

$$\text{Welfare Change per unit of } \partial T_i = \quad (15)$$

$$= W^i(L_i)\,\partial L_i(T_1, T_2)/\partial T_i + W^j(L_j)\,\partial L_j(T_1, T_2)/\partial T_i =$$

$$= [W^i(L_i) - W^j(L_j)]\,\partial L_i(T_1, T_2)/\partial T_i =$$

$$= [W^i(L_i) - W^j(L_j)] / [1/L^1(W^*+T_1)' + 1/L^2(W^*+T_2)'] =$$

$$= (T_i - T_j) / [1/L^1(W^*+T_1)' + 1/L^2(W^*+T_2)']$$

Provided $T_i > T_j$, the welfare effect of the increase in the tax is negative. An unilateral rise in T_i will move resources away from region i – and farther away from the allocation that maximizes total welfare, occurring with either equal tax rates or no tax (see below). If T_j is lower than T_i, however, the opposite occurs – then, an unitary increase in the tax rate approximates the two tax rates and labor allocation to the efficient outcome.

Let there be a uniform fiscal treatment across regions such that $T_i = T$. Then:

$$\partial W^*/\partial T = -1 < 0 \quad (16)$$

Any tax rate change is completely passed on to the net wage. This implies that a change in the tax rate will have no impact on local employment or total welfare.

Summarizing:

Proposition 1: 1.1. Under free market, the usual dualistic model will result in equalization of net wages across regions or sectors and there will be no unemployment.

1.2. Any labor tax rate increase depresses the equilibrium after-tax wage.

1.3. An unilateral increase in the tax rate will decrease local employment and population. It will be welfare enhancing iff the region has lower taxes than the neighbour. The induced labor flow will be larger the higher the slopes of both local labor demands in absolute value.

1.4. A uniform tax rate (due to the inelastic total supply assumption) has no effect on the regional allocation of the labor force – and hence it is compatible with total welfare (product) maximization.

Partial coverage - The perfect mobility case

Assume 3.a, 4.b and 5.a. of section I. The wage in the two sectors differ. In sector 1, the net wage is fixed at level W_1. As the other sector's wage is free, it will decrease till all the labor force is employed – the equilibrium after-tax wage of sector 2, $W_2 < W_1$ for the latter to be a binding restriction:

$$\bar{L}_i = L_i = L^i(W_i + T_i) \quad , \quad i = 1,2 \tag{17}$$

and

$$L^1(W_1 + T_1) + L^2(W_2 + T_2) = \bar{L} \quad \text{or} \tag{18}$$

$$W_2 + T_2 = W^2[\bar{L} - L^1(W_1 + T_1)]$$

An increase in the covered sector tax rate will expel local population while depressing the uncovered sector's net wage:

$$\partial W_2/ \partial T_1 = - L^1(W_1 + T_1)' / L^2(W_2 + T_2)' < 0 \quad (19)$$

$$\partial L_1/\partial T_1 = - \partial L_2/\partial T_1 = L^1(W_1 + T_1)' < 0 \quad (20)$$

Welfare Change per unit of ∂T_1 = (21)

$$= W^1(L_1)\, \partial L_1(T_1, T_2)/\partial T_1 + W^2(L_2)\, \partial L_2(T_1, T_2)/\partial T_1 =$$

$$= [W^1(L_1) - W^2(L_2)]\ \partial L_1(T_1, T_2)/\partial T_1 =$$

$$= [W^1(L_1) - W^2(L_2)]\ L^1(W_1 + T_1)' =$$

$$= [(W_1 + T_1) - (W_2 + T_2)]\ L^1(W_1 + T_1)'$$

The tax will be welfare improving iff

$$W_1 + T_1 < W_2 + T_2 \quad (22)$$

That is possible if gross wages are higher in region 2 (even if net wages are there smaller for the minimum wage in sector 1 to be binding). Then, by raising taxes in region 1, it will be possible to lower sector's 2 net wages enough to boost employment and generate a welfare rise.

A tax on the uncovered sector is passed to the local net wage and has no allocation effects:

$$\partial W_2/ \partial T_2 = -1 < 0 \quad ; \quad \partial L_i/ \partial T_2 = 0,\ i = 1,2. \quad (23)$$

Hence, an optimal global policy to raise a given fiscal revenue will start by taxing region 1 till (22) holds in equality. Henceforth, if possible, both sectors will be taxed preserving it.

Assume that (instead) a uniform tax system must be established. Then:

$$L^1(W_1 + T) + L^2(W_2 + T) = \bar{L} \quad (24)$$

$$\partial W_2 / \partial T = -[L^1(W_1 + T)' + L^2(W_2 + T)'] / L^2(W_2 + T)' < 0 \quad (25)$$

$$\partial L_1/\partial T = -\partial L_2/\partial T = L^1(W_1 + T)' < 0 \quad (26)$$

The implied welfare change will be:

$$\text{Welfare Change per unit of } \partial T = \quad (27)$$

$$= W^1(L_1)\,\partial L_1(T_1, T_2)/\partial T + W^2(L_2)\,\partial L_2(T_1, T_2)/\partial T =$$

$$= [W^1(L_1) - W^2(L_2)]\ \partial L_1(T_1, T_2)/\partial T =$$

$$= [W^1(L_1) - W^2(L_2)]\ L^1(W_1 + T)' =$$

$$= [(W_1 + T) - (W_2 + T)]\ L^1(W_1 + T)' =$$

$$= (W_1 - W_2)\ L^1(W_1 + T)'$$

The tax will never be welfare improving because for a binding minimum wage

$$W_1 > W_2 \quad (28)$$

Summarizing:

Proposition 2: 2.1. In a dualistic model with perfect mobility and institutional wage fixed in one of the sectors, the equilibrium after-tax wage in the second sector is lower than the free market after-tax wage. There will be no unemployment.

2.2. An unilateral increase in the tax rate of the institutionally covered sector will depress the wage of the other sector, which will see its employment increase. It will be welfare improving if and only if the gross wage in the covered sector is lower than in the uncovered one.

2.3. An increase in the tax rate of the uncovered sector will have no impact on the regional allocation of the labor force.

2.4. Under a uniform fiscal system, an increase of the (common) tax rate will relocate the labor force, increasing employment in the uncovered sector (by the same magnitude as a unilateral increase of the tax rate of the uncovered sector would). It will never be welfare improving.

A final comment can be made. At first glance, we could think that taxes would have the same effect as an institutionally set wage. As we see, they do not, even in this simple structure – where total supply does not respond to wages -, provided that the sector's net wage is free. And they have different implications not due to distributional considerations, which we discarded, but due to the generation of different regional or sector employment allocation.

Partial coverage and imperfect mobility - The Harris-Todaro Model

Assume 3.b, 4.b and 5.a. of section I. The wage in the two sectors differ. In sector 1, the net wage is fixed at level W_1.

As the other sector's wage is free, it will decrease till all the local labor force is employed:

$$\bar{L}_2 = L_2 = L^2(W_2 + T_2) \quad (29)$$

However, to have access to wage W_1, people have to locate there, or to specialize if we are addressing industry rather than regional affiliation – implying that unemployment will be generated in the region. There will be labor force flows till

$$W_1 \times \text{ Probability of Employment in region 1} = W_2 \quad (30)$$

That is, in the long run we expect that:

$$W_1 L^1(W_1+T_1) / \bar{L}_1 = W_1 \{L^1(W_1+T_1) / [\bar{L} - L^2(W_2+T_2)]\} = W_2 \quad (31)$$

$$W_1 L^1(W_1+T_1) = W_2 [\bar{L} - L^2(W_2+T_2)] = W_2 \bar{L} - W_2 L^2(W_2+T_2) \quad (32)$$

Consider a change in the tax rate applied to employment in sector 1. Then:

$$\partial W_2 / \partial T_1 = W_1 L^1(W_1+T_1)' / [\bar{L} - L^2(W_2+T_2) - W_2 L^2(W_2+T_2)'] < 0 \quad (33)$$

$\partial L_1/\partial T_1 = L^1(W_1+T_1)'$; $\partial L_2/\partial T_1 = W_1 \; L^2(W_2+T_2)' \; L^1(W_1+T_1)' /$ (34)

$/ [\bar{L} - L^2(W_2+T_2) - W_2 \, L^2(W_2+T_2)'] > 0$

Then, the effect on total unemployment, $U = \bar{L} - L^1(W_1+T_1) - L^2(W_2+T_2)$, is:

$\partial U/\partial T_1 = - L^1(W_1+T_1)' \{1 + W_1 \, L^2(W_2+T_2)' /$ (35)

$/ [\bar{L}-L^2(W_2+T_2)-W_2 \, L^2(W_2+T_2)']\}$ (> 0 if but not only if W_1 is close to W_2)

An increase in the covered sector tax rate will necessarily depress the uncovered sector's wage and raise its employment. We note, once again but now under the H.-T. scenario, that – because we assume that workers respond to after-tax income – the effect does not coincide with the impact of an increase in the covered sector's wage rate [2].

Welfare Change per unit of ∂T_1 = (36)

$= W^1(L_1) \, \partial L_1(T_1, T_2)/\partial T_1 + W^2(L_2) \, \partial L_2(T_1, T_2)/\partial T_1 =$

$= W^1(L_1) \; L^1(W_1 + T_1)' + W^2(L_2) \; W_1 \; L^2(W_2+T_2)' \; L^1(W_1+T_1)' /$

[2] For a direct contrast, see Martins (1996) for example.

$$/ \ [\bar{L} - L^2(W_2+T_2) - W_2 \, L^2(W_2+T_2)'] =$$

$$= \{W^1(L_1) \ L^1(W_1 + T_1)' \, [\bar{L} - L^2(W_2 + T_2)] \ +$$

$$+ \ [W_1 \, W^2(L_2) \ - \ W^1(L_1) \, W_2] \, L^2(W_2 + T_2)' \, L^1(W_1 + T_1)'\} \, /$$

$$/ \, [\bar{L} - L^2(W_2 + T_2) - W_2 \, L^2(W_2 + T_2)']$$

The first term is negative. Yet, the second may be positive. For $T_1 = T_2 = 0$, the expression solves for the first term only:

$$W^1(L_1) \ L^1(W_1 + T_1)' \, [\bar{L} - L^2(W_2 + T_2)] \, / \, [\bar{L} - L^2(W_2 + T_2) - W_2 \, L^2(W_2 + T_2)'] \ < \ 0$$

The effect of the tax on welfare is then unambiguously negative.

(Interestingly, Srinivasan & Bhagwati (1975), analysing the impact of a wage subsidy to the institutional sector on a version of the H-T. model find it welfare enhancing. However, their setting differs from ours, once, by imposing redistribution and production of an homogeneous good, we discard – unlike them – any implicit terms-of-trade effect.)

If it is the uncovered sector tax that rises:

$$\partial W_2 \ / \ \partial T_2 \ = \ W_2 \ L^2(W_2+T_2)' \ / \ [\, \bar{L} \ - \ L^2(W_2+T_2) \ - \ W_2 \ L^2(W_2+T_2)'] < 0 \qquad (37)$$

$$\partial L_1/\partial T_2 = 0 \ \ ; \ \ \partial L_2/\partial T_2 \ = \ L^2(W_2 + T_2)' \ \ [\bar{L} - L^2(W_2+T_2)] \, / \qquad (38)$$

$$/ [\bar{L} - L^2(W_2+T_2) - W_2 L^2(W_2+T_2)'] < 0$$

$$\partial U/\partial T_2 = - \partial L_2/\partial T_2 > 0 \tag{39}$$

Welfare Change per unit of ∂T_2 = (40)

$$= W^1(L_1)\,\partial L_1(T_1, T_2)/\partial T_2 + W^2(L_2)\,\partial L_2(T_1, T_2)/\partial T_2 =$$

$$= W^2(L_2)\,\partial L_2(T_1, T_2)/\partial T_2 =$$

$$= W^2(L_2)\; L^2(W_2 + T_2)'\; [\bar{L} - L^2(W_2+T_2)] /$$

$$/ [\bar{L} - L^2(W_2+T_2) - W_2 L^2(W_2+T_2)'] < 0$$

The welfare change of a rise in T_2 will be negative.

Supposing now a uniform tax system and (32) becoming:

$$W_1\, L^1(W_1+T) = W_2\, [\bar{L} - L^2(W_2+T)] = W_2\, \bar{L} - W_2\, L^2(W_2+T) \tag{41}$$

$$\partial W_2 / \partial T = [W_1\, L^1(W_1+T)' + W_2\, L^2(W_2+T)'] / \tag{42}$$

$$[\bar{L} - L^2(W_2 + T) - W_2\, L^2(W_2 + T)'] < 0$$

$$\partial L_1/\partial T = L^1(W_1 + T)' < 0 \tag{43}$$

$$\partial L_2/\partial T = L^2(W_2+T)'\, [\bar{L} - L^2(W_2 + T) + W_1\, L^1(W_1 + T)'] /$$

$$/ [\bar{L} - L^2(W_2 + T) - W_2\, L^2(W_2 + T)']$$

$\partial L_2/\partial T < 0$ iff $|\varepsilon^1| < (W_1 + T) / W_2$, where ε^i denotes $L^1(W_1 + T)' (W_1 + T) / L^1(W_1 + T)$.

$$\partial U/\partial T = -\{[L^1(W_1+T)' + L^2(W_2+T)'] [\bar{L} - L^2(W_2 + T)] + \{L^1(W_1 + T)' L^2(W_2+T)' (W_1 - W_2)\} / \quad (44)$$

$$/ [\bar{L}-L^2(W_2+T)-W_2 L^2(W_2+T)']$$ (> 0 if but not only if W_1 is close to W_2)

We can assess the induced welfare change after a tax movement by:

Welfare Change per unit of ∂T = (45)

$$= W^1(L_1)\, \partial L_1(T_1, T_2)/\partial T + W^2(L_2)\, \partial L_2(T_1, T_2)/\partial T =$$

$$= W^1(L_1)\; L^1(W_1 + T)' + W^2(L_2)\; L^2(W_2 + T)' [\bar{L} - L^2(W_2 + T) + W_1 L^1(W_1 + T)'] /$$

$$/ [\bar{L} - L^2(W_2 + T) - W_2 L^2(W_2 + T)']$$

Provided $W_1 L^1(W_1 + T)'$ is negligible, the effect will be negative.

Proposition 3: 3.1. Consider a dualistic model with no mobility. The equilibrium after-tax wage in the second sector may be higher or lower than the free market equilibrium, in which there will be unemployment in the institutional sector or urban region.

3.2. An unilateral increase in the tax rate of the institutionally covered sector will depress the wage of the other sector, which will see its employment increase. Total unemployment will likely increase.

3.3. An increase in the tax rate of the uncovered sector will decrease its employment - and local population - and increase total unemployment by the same amount.

3.4. Under a uniform fiscal system, an increase of the (common) tax rate will relocate the labor force, increasing (decreasing) employment in the uncovered sector if the elasticity of demand of the covered sector is high (low) - higher (lower) than the ratio between the gross wage of the covered sector and the after-tax wage of the uncovered one. Total unemployment will likely increase.

3.5. In general, a raise in a tax rate will be welfare detracting.

Multiple or global coverage under imperfect mobility - The Bhagwati-Hamada Model

Assume 3.b, 4.b. and 5.b. of section I. The wage in both sectors are fixed at level's W_i, i =1,2. One can see this same (technically speaking) scenario in, for example, Bhagwati & Hamada (1974).

Then, local employment, being demand determined:

$$L_i = L^i(W_i + T_i) \tag{46}$$

Let U_i be the local unemployment in region i, i.e.:

$$U_i = \bar{L}_i - L_i \tag{47}$$

Denote by u_i the unemployment rate in sector/region i. Define:

$$u_i = U_i / \bar{L}_i = 1 - L_i / \bar{L}_i \tag{48}$$

The equilibrium condition will establish equalization of expected income in both sectors:

$$(1 - u_1) W_1 = (1 - u_2) W_2 \tag{49}$$

that is, equilibrium is defined by:

$$W_1 L^1(W_1 + T_1) / \bar{L}_1 = W_2 L^2(W_2 + T_2) / \bar{L}_2 \tag{50}$$

In equilibrium, the average net wage in the economy, $[W_1 L^1(W_1 + T_1) / \bar{L}_1] \bar{L}_1 / \bar{L} + [W_2 L^2(W_2 + T_2) / \bar{L}_2] \bar{L}_2 / \bar{L}$, is equal to the expected wage in each sector, $W_i L^i(W_i) / \bar{L}_i$.

One can re-write condition (50) as:

$$W_1 L^1(W_1 + T_1) (\bar{L} - \bar{L}_1) = W_2 L^2(W_2 + T_2) \bar{L}_1 \tag{51}$$

$$\partial\bar{L}_i/\partial T_i = - \partial\bar{L}_j/\partial T_i = \bar{L}_j W_i L^i(W_i + T_i)' / \; / [W_1 L^1(W_1 + T_1) + W_2 L^2(W_2 + T_2)] < 0 \tag{52}$$

As $W_j\ L^j(W_j + T_j)$ is fixed, this also implies that equilibrium expected after-tax wage $W_2\ L^2(W_2 + T_2) / \bar{L}_2 = W_1\ L^1(W_1 + T_1) / \bar{L}_1$ will decrease with T_i. As both W_i's are fixed, the local unemployment rate in both regions will always increase with either T_i.

$$\text{Welfare Change per unit of } \partial T_i = \tag{53}$$

$$= W^i(L_i)\ \partial L_i(T_1, T_2)/\partial T_i = (W_i + T_i)\ L^i(W_i + T_i)' < 0$$

This would be the standard effect of a raise in taxes (as of the institutional wage) in a unionised single-sector economy that has inelastic supply. $W_i + T_i = W^i(L_i)$ and (expected to) equals the value of marginal product of labor $P\ f^{i\prime}(L_i)$ [3], where $f^i(L_i)$ denotes the local production function of region i. On the other hand, $L^i(W_i + T_i)' = 1 / W^i(L_i)'$ and:

$$\text{Welfare Change per unit of } \partial T_i = (W_i + T_i) / W^i(L_i)' = \tag{54}$$

$$= W^i(L_i) / W^i(L_i)' =$$

$$= [P\ f^{i\prime}(L_i)] / [P\ f^{i\prime\prime}(L_i)] = f^{i\prime}(L_i)] / f^{i\prime\prime}(L_i)$$

The welfare loss will be larger the less concave is the local production function – the less negative $f^{i\prime\prime}(L_i)$. For small tax

[3] Implicitly, we could fix P = 1, once the only good in the economy is income or product.

changes, it will decrease with (minus) $- f^{i\prime\prime}(L_i) / f^{i\prime}(L_i)$ and we recall that $- f^{i\prime\prime}(L_i) / f^{i\prime}(L_i)$ is the Arrow-Pratt measure of absolute risk-aversion – of concavity embedded in a function $f^i(L_i)$. It is (here) equal to the inverse of the absolute value of the inverse [4] semi-elasticity of labor demand, $| W^i(L_i) / W^i(L_i)' | = | W^i(L_i) L^i(W_i + T_i)' |$.

Consider an uniform tax system. Then:

$$W_1 L^1(W_1 + T) (\bar{L} - \bar{L}_1) = W_2 L^2(W_2 + T) \bar{L}_1 \tag{55}$$

$$\partial\bar{L}_i/\partial T = - \partial\bar{L}_j/\partial T = [\bar{L}_j W_i L^i(W_i + T)' - \bar{L}_i W_j L^j(W_j + T)'] / \tag{56}$$

$$/ [W_1 L^1(W_1 + T) + W_2 L^2(W_2 + T)] < 0$$

After an increase in the tax rate, there will be an inflow of population to region i iff:

$$\bar{L}_j W_i L^i(W_i + T)' > \bar{L}_i W_j L^j(W_j + T)' \tag{57}$$

or $- L^i(W_i + T)' / L^i(W_i + T) < - L^j(W_j + T)' / L^j(W_j + T)$

[4] The term "inverse" may not be appropriate. The concept measures the change in labor demand (and employment) induced per unitary proportional increase of the gross wage rate.

There will be an inflow of population to the region of lower (absolute) semi-elasticity of labor demand.

$$\text{Welfare Change per unit of } \partial T = \tag{58}$$

$$= (W_1 + T)\, L^{1}(W_1 + T)' + (W_2 + T)\, L^{2}(W_2 + T)' < 0$$

Proposition 4: 4.1. With multiple coverage, the increase in a or both tax rates will increase total unemployment and both local unemployment rates.

4.2. An unilateral increase in the tax rate of one of the regions will generate an outflow of migrants to the other region, where the unemployment rate will (also) rise.

4.3. Under a uniform fiscal system, an increase of the (common) tax rate will relocate the labor force, increasing (decreasing) residence in the sector of lower absolute value of the semi-elasticity of labor demand.

4.4. The increase in taxes will always generate welfare losses – proportional to the gross wage and to the slope of the demand of the sector where the tax rate rises, decreasing with the concavity of the underlying production function - increasing with the absolute size of the inverse semi-elasticity of local labor demand.

Size restrictions in the sectors

In this section we want to quantify the effects of several changes in the two-sector model with institutional wage fixed in sector 1 but with size restriction in the areas. These could reproduce regional congestion, but not necessarily national boundaries. They are expected to independentize tax effects across regions.

We always assume 3.b. and 4.b. of section I. We distinguish two cases:

Model A:

Region 1 has a limited housing capacity, or there are barriers to membership or affiliation in region 1 (say, "insiders" limit entry). We add assumption:

5.c. Wage in the second sector is market determined and entry location restrictions in region 1 place an upper bound of $\bar{L}_1^*$ on the amount of people that can actually live there.

If the restriction is binding in equilibrium, supply in the second sector will be also fixed:

$$\bar{L}_2 = \bar{L} - \bar{L}_1^* \tag{59}$$

and the wage in the second sector:

$$W_2 = W^2(\bar{L} - \bar{L}_1^*) - T_2 \tag{60}$$

It must be the case that $\bar{L}_1^*$ is smaller than the equilibrium local supply in the institutional sector generated in the Harris-Todaro framework. As long as that condition holds, dynamics of this scenario have some of the same properties of the partial coverage - perfect mobility case.

It is straightforward to show that the tax on the uncovered sector has no allocation effects and implies no welfare loss:

$$\partial W_2/\partial T_2 = -1 \;\; ; \;\; \partial L_2/\partial T_2 = 0 \tag{61}$$

The impact of the tax on region 1 is totally reflected in local (un)employment:

$$\partial L_1/\partial T_1 = L^1(W_1 + T_1)' \tag{62}$$

A uniform tax will hurt only – and according to (62) – sector 1's employment. The welfare loss of an increase in the tax on the covered sector always results in:

Welfare Change per unit of ∂T_1 (or ∂T) = (63)

$$= (W_1 + T_1)\ L^1(W_1 + T_1)' < 0$$

Proposition 5: 5.1. In a dualistic model with housing or membership restrictions and institutional wage fixed in one of the sectors, the equilibrium after-tax wage in the uncovered sector is higher than for the perfect mobility case. There will be unemployment but less than in the H.-T. framework.

5.2. Given the mobility barrier, an increase of a tax rate will have mainly local effects: no effect for the employment of the uncovered sector; a reduction in the covered sector employment, of local and total welfare when its tax rate rises.

Model B:

The traditional sector has a limited ability of employment generation, say land (and land productivity) is fixed or limited; or there are employment quotas in the region. We consider assumption:

5.d. Wage in second sector is demand determined but there is a(n exogenous) limit of access to employment in the sector given by L_2^*.

Then, the wage in the second sector is determined by:

$$L_2^* = L^2(W_2 + T_2) \quad \text{or} \quad W_2 = W^2(L_2^*) - T_2 \tag{64}$$

If employment in sector 2 is fixed, equilibrium is guaranteed by population flows which will be generated until:

$$W_1 L^1(W_1 + T_1) / \bar{L}_1 = L_2^* [W^2(L_2^*) - T_2] / \bar{L}_2 \tag{65}$$

We will have the same type of scenario as in B.-H. – unemployment in both sectors. However, the fiscal implications differ.

As employment is fixed – rationed - in sector 2, any change in the local tax is totally absorbed by falls in net wages:

$$\partial W_2/\partial T_2 = -1 \ ; \ \partial L_2/\partial T_2 = 0 \tag{66}$$

Population will flow to region 1 where local unemployment increases.

$$\partial \bar{L}_1/\partial T_2 = -\partial \bar{L}_2/\partial T_2 = \bar{L}_1 L_2^* / [W_1 L^1(W_1 + T_1) + W_2 L_2^*] > 0 \tag{67}$$

An unilateral increase in taxes of region 1 has the same effect as in the B.-H. structure:

$$\partial \bar{L}_1/\partial T_1 = -\partial \bar{L}_2/\partial T_1 = \bar{L}_2 W_1 L^1(W_1 + T_1)' / \tag{68}$$

$$/ [W_1 L^1(W_1 + T_1) + W_2 L_2^*] < 0$$

Welfare Change per unit of ∂T_1 (or ∂T) = (69)

$$= (W_1 + T_1)\ L^1(W_1 + T_1)' < 0$$

A uniform system, under which:

$$W_1\ L^1(W_1 + T) / \bar{L}_1 = L_2^* [W^2(L_2^*) - T] / \bar{L}_2 \quad (70)$$

implies that (66) still holds for sector 2 and:

$$\partial \bar{L}_1/\partial T = - \partial \bar{L}_2/\partial T = [\bar{L}_1\ L_2^* + W_1\ L^1(W_1 + T)'\ \bar{L}_2] / \quad (71)$$
$$/ [W_1\ L^1(W_1 + T_1) + W_2\ L_2^*]$$

$$\partial \bar{L}_1/\partial T > 0 \text{ iff } - L^1(W_1{+}T)'\ (W_1{+}T)/L^1(W_1{+}T) = |\ \varepsilon^1\ | < (W_1{+}T) / W_2 \quad (72)$$

Proposition 6: With a employment-congested sector, we arrive at equilibrium conditions similar to B.H. structure – but not to the same conclusions about the response to fiscal changes.

6.1. An unilateral increase in the tax rate of the institutionally covered sector will depress its – and total – employment, local and total welfare, generating an outflow of population to the congested sector.

6.2. An increase in the tax rate of the congested sector will have no impact on employment or welfare but will imply an outflow of population from the congested sector, where after-tax wages fall.

6.3. Under a uniform fiscal system, an increase of the (common) tax rate will relocate the labor force, increasing (decreasing) residence in the congested sector if the elasticity of demand of the covered sector is high (low) - higher (lower) than the ratio between the gross wage of the covered sector and the after-tax wage of the congested one. Total unemployment will increase – by the amount that employment in the covered sector decreases. Welfare decreases.

Summary and conclusions

In the presence of competitive labor markets and perfect mobility across regions – or sectors -, uniform taxation offers first-best allocations. Moreover, as we always assume redistribution and inelastic total factor supply, uniform taxation under such circumstances implies no deadweight loss, being completely "neutral". Such result (even if not neutrality) – frequently encountered in the public finance literature [5] – was found to fall when the first condition fails, whether the second holds or not.

On the one hand, it was demonstrated that a unit tax on a minimum wage covered / unionised sector may be total welfare improving if the region communicates with an uncovered sector one. In general, the possibility requires differential taxation, heavier on the uncovered than on the covered sector.

On the other, with less than perfect mobility – hence with some unemployment in the economy – taxing the uncovered sector will more likely be detrimental to total welfare.

With two covered sectors, the welfare loss due to taxation is (approximately) inversely related to the concavity of the underlying production technology of the sector where the tax is levied – directly related to the (absolute) magnitude of the inverse semi-elasticity of the sector's labor demand.

[5] For head taxes, in Mieszkowski & Zodrow (1989); in Gordon (1983), p.583, appraising taxation of a mobile commodity such as capital.

The population outflow predictions after a unilateral increase in one of the tax rates were the expected ones for all scenarios: away from the increasingly taxed location. Interestingly, whenever a uniform tax system is required, an increase in the common tax rate under imperfect mobility will shift the labor force to the (un)covered sector if the wage-regulated demand elasticity is (high) low in absolute value; if both sectors are covered, to the region of lower (absolute) semi-elasticity of labor demand.

Under sector/region size or affiliation restrictions, mobility across regions is somehow lost. Then, an uncovered sector is always cushioned from fiscal welfare losses. A covered sector will always experience them after the own tax rate increases.

References

Atkinson, A.B., & Stiglitz, J.E. (1980). *Lectures on Public Economics.* New York: McGraw-Hill.

Bhagwati, J.N. (1982) Introduction. *Journal of Public Economics.* 18(3), 285-289. doi. 10.1016/0047-2727(82)90033-0

Bhagwati, J.N. & Hamada, K. (1974). The Brain Drain, international integration of markets for professionals and unemployment: A theoretical analysis. *Journal of Development Economics.* 1(1), 19-42. doi. 10.1016/0304-3878(74)90020-0

Bhagwati, J.N., & Hamada, K. (1982). Tax policy in the presence of emigration. *Journal of Public Economics.* 18(3), 291-317. doi. 10.1016/0047-2727(82)90034-2

Bhattacharya, P.C. (1993). Rural-urban migration in economic development. *Journal of Economic Surveys.* 7(3), 243-281. doi. 10.1111/j.1467-6419.1993.tb00167.x

Bosworth, D., Dawkins, P., & Stromback, T. (1996). *The Economics of the Labour Market*. London: Addison Wesley Longman.

Brown, C., Gilroy, C., & Kohen, A. (1982). The effect of the minimum wage on employment and unemployment. *Journal of Economic Literature.* 20(2), 487-528.

Fields, G.S. (1989). On-the-job search in a labor market model: Ex ante choices and ex post outcomes. *Journal of Development Economics.* 30(1), 159-178. doi. 10.1016/0304-3878(89)90056-4

Gordon, R.H. (1983). An optimal taxation approach to fiscal federalism. *Quarterly Journal of Economics.* 98(4), 567-586. doi. 10.2307/1881778

Harris, J.R., & Todaro, M.P. (1970). Migration, unemployment and development: A two-sector analysis. *American Economic Review.* 60(1), 126-42.

Johnson, H.G., & Mieszkowski, P. (1970). The effects of unionisation on the distribution of income: A general equilibrium approach. *Quarterly Journal of Economics.* 84(4), 539-561. doi. 10.2307/1880839

Layard, R., & Walters, A.A. (1978). *Microeconomic Theory.* New York: McGraw-Hill.

Martins, A.P. (1996). Migration and mobility: An analytical survey with some extensions. Working Paper, N.72.96. Lisboa, Universidade Católica Portuguesa.

McDonald, I.M., & Solow, R.M. (1985). Wages and employment in a segmented labor market. *Quarterly Journal of Economics.* 100(3), 1115-1141. doi. 10.2307/1885677

McNabb, R., & Ryan, P. (1990). Segmented labour markets. In *Current Issues in Labour Economics.* Edited by D. Sapsford & Z. Tzannatos. Houndmills: Macmillan.

Mieszkowski, P., & Zodrow, G.R. (1989). Taxation and the Tiebout Model: the differential effects of head taxes, taxes on land rents, and property taxes. *Journal of Economic Literature*. 27(3), 1098-1146.
Mincer, J. (1976). Unemployment effects of minimum wages. *Journal of Political Economy*. 84(4), S87-S104. doi. 10.1086/260534
Musgrave, R.A., & Musgrave, P.B. (1976). *Public Finance in Theory and Practice*. 2nd Edition. Tokyo: McGraw-Hill.
Oates, W.E. (1999). An essay on fiscal federalism. *Journal of Economic Literature*. 37, 1120-1149.
Saint-Paul, G. (1996). *Dual Labour Markets: A Macroeconomic Perspective*. The MIT Press.
Srinivasan, T.N., & J.N. Bhagwati. (1975). Alternative policy rankings in a large, open economy with sector-specific minimum wages. *Journal of Economic Theory*. 11(3), 356-371. doi. 10.1016/0022-0531(75)90024-1
Tiebout, C.M. (1956). A pure theory of local expenditures. *Journal of Political Economy*. 64(5), 416-424. doi. 10.1086/257839
Todaro, M.P. (1969). A model of labor migration and urban unemployment in less developed countries. *American Economic Review*. 59(1), 138-48.
Tresch, R.W. (1981). *Public Finance: A Normative Theory*. Plano: Business Publications.

3 Union duopoly with homogeneous labor: The effect of membership and employment constraints

Introduction

This chapterconsiders a two-union closed-shop equilibrium and studies the effect of quantity or employment constraints that generate corner solutions in the labor market equilibrium. These constraints may be legally induced; they can be used as tools for employment-enhancing policies in strongly unionized (or corporatized) economies, professions and industries.

The multiple union case has been previously studied in the literature. Oswald (1979) [1] departs from unions with price competition strategies - framework also used by Gylfason & Lindbeck (1984a) and (1984b) [2] - and derives the properties of the Cournot [3]-Nash [4] equilibrium, comparing it with the Stackelberg [5] one and even describing efficient cooperation [6] between unions. He assumes heterogeneous labor with substitutability between workers. Martins & Coimbra (1997 and 1997a) analyze the equilibrium solutions for homogeneous workers in a market with two and n unions; as Hart´s (1982) syndicates, unions employ quantity rather than price strategies. The assumption applies whenever unions can influence lay-off regulation – as is the case for Portugal, for example.

In the standard imperfect competition framework, pure or unqualified quantity restrictions leading to corner solutions have not been assigned particular interest. However, two problems can be addressed in the union case. Both introduce kinks and one discontinuities in the affected union's reaction function.

The first case we address is insufficient number of members of one of the unions to achieve the interior solution. The effect of a membership constraint has been studied in a monopoly union framework by Carruth & Oswald (1987) in a different context - they model the restriction as embedded in the union's utility function and analyze the possible implications for the hiring of outsiders. Instead, we focus on a two union scenario, and start by general utility functions in a closed-shop. If one of the unions reaches the bound, the other may behave as a monopolist with respect to residual demand; and we suggest the possible advantage for the "larger" union to decrease her own employed members and push the other to the membership constraint.

Employment ceilings (limits of access to employment) may have an immediate application in the study of occupational licensure, the analysis reflecting Friedman’s (1962,1982) historical exposition on its effects: if licensing may provide consumer protection in an imperfectly informed world, it can also be used by an incumbent group to secure monopoly (oligopoly, monopolistic) power over a specific market - the argument is mathematical, yet simply, developed below. The bound can be seen to work through the recognition of diplomas or certification requirements for, say, an immigrant group. One can offer a practical application of this environment in the Portuguese dentist market: Brasilian dentists co-existed in the market but illegaly - only recently have they achieved recognition – with recognized Portuguese dentists. The same can be said about formal and parallel medicine. In such cases, professional associations, take the role of “our” union – and usually fight for barriers to foreign diploma´s recognition.

The other case is the existence of union representative rules (laws) or minimum employment requirements. Say the union to be considered legal must exhibit a minimum number of employed members. The hypothesis, which is realistic, was introduced in the n-union framework by Martins & Coimbra (1997a) to explain, or rather, limit union formation; for example, Portuguese labor law requires a minimum of 10% of target workers or 2000 workers present for a founding assembly to function [7]. The existence of such laws, again, may lead to corner solutions; or may elicit entry-deterrence - "limit output" - practices by the incumbent [8]. The research in this respect shows analogies with the capacity constraint literature for the product market, such as Dixit's (1979) problem [9], with entry-deterrence working in the union framework through overemployment of the incumbent.

The implications of the two constraints are studied for the Cournot-Nash equilibrium, the Stackelberg solution and for cooperative unions. Some of the strategic behavior analyzed is more plausible in Stackelberg than in the Cournot-Nash framework - for example, employment-pushing and entry-deterrence practices. Others may arise in both situations.

This research is mainly theoretical; it has empirical relevance in the understanding of the behavior of industry unions, in the presence of representative requirement rules. The closed-shop environment reproduces the scenario where bargaining agreements are extended to non-unionized workers – as they usually are in Portugal: if unionisation was around 30% in the early nineties, the collective bargaining coverage rate – number of workers covered by collective agreements as a percentage of wage and salary earners - was almost 80%, according to OECD [10]; the proportion of non-self-employed covered seems to have remained stable in the second half of the decade [11]. The duopoly setting applies, for example, to Portugal, where some industries or professions are represented by two unions. At the macro or aggregate level, in the first half of the nineties, two union confederations [12] represented 88% of Portuguese unionised workers, the behavior of which this research may also address; insufficient membership, for example, may have conditioned the behavior of the newer confederation [13] in its early stages; and may have affected (formally, the corresponding restriction can become active after more or less sudden decreases in unionised affiliates) the older confederation, that lost, on average, 46% of its members from 1979-84 to 1991-95 [14].

Interior solution results and notation are summarized in section II. Insufficient membership for internal solutions is dealt with in section III. Implications of the existence of a minimum employed members requirement is advanced in section IV. The exposition ends with a brief summary in section V, which includes tables with the main analytical results along with those for an example.

Union duopoly: Interior solutions [15]

Consider that we have two unions, 1 and 2. Let L_i be the employed members of union i, and W the wage rate. The labor demand in the market is given by:

$$L_1 + L_2 = L(W), \quad \text{or} \tag{1}$$

$$W = W(L_1 + L_2) = P F_L(L_1 + L_2) \tag{2}$$

Workers are perfect substitutes and the wage set by firms will be extended to all workers - or firms will equate marginal product for the two types of workers, or will only hire workers of lower wage.

Assume

a) each union has utility function of the general form $U^i(L_i, W)$, increasing in the arguments and quasi-concave, for which U^i_L / U^i_W - the marginal rate of substitution between employment and wage - decrease with L_i and increases with W.

b) the demand function is given by (1), decreasing in W, coming from maximization (in $L = L_1 + L_2$) of the (aggregate) profit function ◎(L,W). Therefore, (2) establishes the value of the marginal product of labor, equal for both types of workers[16].

c) a sort of closed-shop setting, i.e., the firm(s) can only hire unionized workers, either from union 1 or 2.

Cournot-Nash duopoly

Each union maximizes

$$\max_{L_i, W} U^i(L_i, W) \quad (3)$$

s.t.: $L_1 + L_2 = L(W)$ or $W = W(L_1 + L_2) = P\, F_L(L_1 + L_2)$

or, alternatively:

$$\max_{L_i} U^i[L_i, W(L_1 + L_2)] \quad , \quad i=1,2, \quad (4)$$

F.O.C. imply

$$U^i_L + U^i_W \; W_L = 0 \quad , \quad i = 1,2 \qquad (5)$$

(5) establishes the optimal policy of union i given union j's employment strategy, i.e., union i's reaction function:

$$L_i = R^i(L_j) \qquad i = 1, 2\,; \;\; j = 2, 1 \qquad (6)$$

The Nash-Cournot equilibrium will be described by the two equations:

$$U^1_L / U^1_W = U^2_L / U^2_W = -W_L \quad \text{or}$$

(7)

$$U^1_W / U^1_L = U^2_W / U^2_L = -L_W$$

and the demand function.

Alternatively, the equilibrium labor market outcome is defined by the two equations (and labor demand which defines the wage):

$$L_1 = R^1(L_2) \qquad \text{and} \qquad L_2 = R^2(L_1) \qquad (8)$$

that is:

$$L_1 = R^1[R^2(L_1)] \qquad \text{and / or} \qquad L_2 = R^2[R^1(L_2)] \qquad (9)$$

As in the standard imperfect competition problem, (static) stability [17] is only achieved iff - for negatively sloped reaction functions:

$$| \, dR^2 / dL_1 \, | \circledcirc 1 / | \, dR^1 / dL_2 \, | \qquad (10)$$

This will be satisfied if both unions' reaction functions have slope smaller than 1 in absolute value, i.e., $| dR^2 / dL_1 | \leq 1$ and $| dR^1 / dL_2 | \leq 1$. Therefore, we will assume continuous, smooth and well-behaved reaction functions $R^i(L_j)$ - derived as in (6) -, and that:

$$-1 < dR^i(L_j) / dL_j < 0 \tag{11}$$

Given that we have only two unions, we can represent the equilibrium solution in the space (L_1, L_2) - analogous to the conventional product market solution for quantities of an homogeneous good sold by two firms. This is presented in Fig. 1 below. PN is union 1's reaction function and MQ is union 2's. Each union's indifference curve will have the general form:

$$U^i[L_i, W(L_1 + L_2)] = \bar{U}^i \quad , \quad i = 1, 2 \tag{12}$$

Once the reaction functions come from (5), each union's indifference curve has a maximum on its reaction function. The level of utility increases as we approach, on the reaction curve, the union's employment axis; that curve crosses this axis at the monopoly union solution (where the other union's employment is 0).

Both reaction functions are negatively sloped, obeying condition (10). They cross at point A, the Cournot-Nash equilibrium.

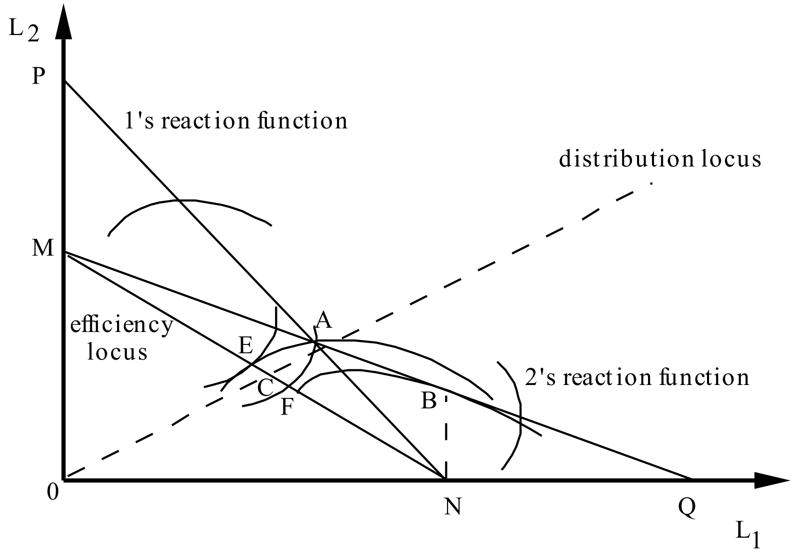

Fig. 1

Stackelberg duopoly

Take now the case where union 1 operates as a leader and union 2 responds as its follower. Then the leader solves:

$$\max_{L_1, L_2} \; U^1[L_1, W(L_1 + L_2)] \tag{12}$$

$$\text{s.t.:} \quad U^2_L + U^2_W W_L = 0 \quad \text{or} \quad L_2 = R^2(L_1)$$

Constructing the Lagrangean, or replacing the restriction in the utility function, and considering F.O.C. we get:

$$U^1_L / U^1_W = (1 + dR^2 / dL_1)\, U^2_L / U^2_W = -(1 + dR^2 / dL_1)\, W_L \tag{14}$$

$$L_2 = R^2(L_1) \quad \text{or} \quad U^2_L / U^2_W = -W_L \tag{15}$$

We can see the Stackelberg solution in Fig. 1. Union 1 picks the point on 2's reaction function which allows her to reach the highest level of utility, i.e., the indifference curve closer to the L_1 axis that touches 2's reaction function. This is point B. Confronting with A, the Cournot-Nash equilibrium, the leader reaches now a higher employment and utility level, and the follower smaller levels, than in the Cournot-Nash solution. Total employment will be larger than in the Cournot solution: the Stackelberg equilibrium, point B, is on 2's reaction function, with slope smaller than one in absolute value, to the right of and below A.

Efficient cooperation between the unions

Assume the unions cooperate efficiently. That is, the two unions maximize the Nash-bargaining problem:

$$\max_{L_1, L_2, W} \; [U^1(L_1, W) - \bar{U}^1]^{\circledcirc} \; [U^2(L_2, W) - \bar{U}^2] \qquad (16)$$

$$\text{s.t.:} \quad L_1 + L_2 = L(W) \quad \text{or} \quad W = W(L_1 + L_2)$$

$\bar{U}^1$ and $\bar{U}^2$ denote the alternative utility of each union in case of no agreement. Eventually, $\bar{U}^i$ would be the utility union i gets in the Cournot game[18]. $\circledcirc$ corresponds to the relative strength of union 1 with respect to union 2 within the coalition[19]. The optimal solution will yield:

$$U^1_W / U^1_L + U^2_W / U^2_L = -L_W \qquad (17)$$

We immediately expect a higher W and a lower L when cooperation between unions is established.

If we replace the demand schedule - $W = W(L_1 + L_2)$ - in (17), this equation defines the "efficiency locus" [20] - a relation between L_1 and L_2 where the utility of one of the unions is maximized given the utility level of the other -, given that employers react on the demand.

From F.O.C., we can also derive:

$$\circledcirc = \{[U^1(L_1, W) - \bar{U}^1] / [U^2(L_2, W) - \bar{U}^2]\} \, (U^2{}_L / U^1{}_L) \qquad (18)$$

Replacing, again, the demand schedule, this equation establishes the "distribution locus" - a relation between L_1 and L_2.

The intersection of the efficiency locus and the distribution locus yields the particular solution of the problem.

We can see the equilibrium, again, in Fig. 1. The efficiency locus is MN, a curve formed by the intersection of the two unions' indifference curves; it should cross the axis of L_i (i=1, 2) at the same point where i's reaction curve does, once this point determines the monopoly solution for union i. If $\bar{U}^i$ is the utility union i gets in the Cournot game, the distribution equation will cross the efficiency locus between points E and F - points on the indifference curves of each union corresponding to their utility level in the Cournot-Nash equilibrium. The efficient bargaining equilibrium is, thus, point C. In general, we expect lower employment than in the previous solutions - distribution between the unions depending on the location of the distribution locus, and, therefore, also on $\circledcirc$.

Insufficient membership

Assume that a particular union i has an exogenously fixed number of members M_i which is smaller than the equilibrium level of employment corresponding to the interior solution. What will this imply for the labor market outcome? Will the other (presumably larger) union have any advantage in restricting its own quantity in order to make the former reach its bound, i.e., the employment ceiling?

Cournot duopoly

1. Consider that the two unions are Cournot duopolists. Then, we know that for an interior solution:

$$L_i = R^i(L_j) \quad i=1,2, \quad j=2,1 \tag{19}$$

Alternatively, the equilibrium L_i^* satisfies:

$$L_i^* = R^i(R^j(L_i^*)) \quad i=1,2, \quad j=2,1 \tag{20}$$

2. If $R^2(R^1(L_2^*)) > M_2$, then:

$$L_2 = M_2 \qquad \text{and} \qquad L_1 = R^1(M_2) \tag{21}$$

This would seem to require that:

$$R^2(R^1(M_2)) > M_2 \tag{22}$$

That if $R^2(R^1(L_2^*)) > M_2$, then $R^2(R^1(M_2)) > M_2$, is proven in the Appendix.

Being the reaction functions negatively sloped, union 1 will benefit, to some extent, from the unfulfilled demand of the other union. At the "corner" equilibrium:

$$U^2_L + U^2_W W_L > 0 \quad \text{at} \quad L_2 = M_2 \tag{23}$$

and

$$U^1_L + U^1_W W_L = 0 \quad \text{at} \quad L_2 = M_2 \,, \quad \text{or} \quad L_1 = R^1(M_2) \tag{24}$$

Union 1 will have higher employment - once reaction functions are negatively sloped - and attain higher utility level than if 2's members allowed the interior solution.

We can see the new solution in Fig. 2. Graphically, 2's reaction function, MN without restriction, now becomes M_2BN, being M_2 the number of members of union 2 and smaller than employment of union 2 in the unrestricted Cournot equilibrium, point A. With insufficient membership of union 2, we go to point C, on 1's reaction function.

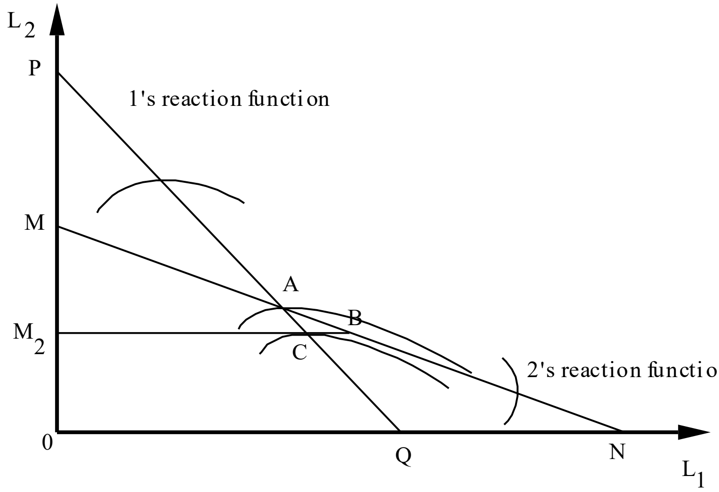

Figure 2.

3. In a corner solution for union i:

$$L = M_2 + R^1(M_2) \tag{25}$$

and

$$W = W(M_2 + R^1(M_2)) \tag{26}$$

An increase in membership of union 2, M_2, will have an impact such that:

$$dL/dM_2 = 1 + dR^1/dL_2(M_2) \tag{27}$$

Total quantity will increase (equilibrium wage will decrease) with membership of union i as long as:

$$dR^1(M_2)/dL_2 > -1 \tag{28}$$

This is included in condition (11).

As M_2 increases, we tend to the Cournot solution - as we can see in Fig. 2. Having 1's the reaction functions slope smaller than 1 in absolute value, the corner solution will - given (27)-(28) -, imply a smaller total employment and higher wage than the interior solution.

Proposition 1: If membership is not sufficient for an internal solution in a

Cournot duopoly:

1. the small union will employ all its members.

2. the large union will behave as a monopolist with respect to the residual

demand. It will have higher employment and attain a higher utility level

than in the interior solution.

3. Total employment will be smaller and the wage higher than in the

interior solution.

4. An increase in membership of the small union will decrease the other

union's employment, increase total employment and decrease the wage.

Stackelberg equilibrium

Consider now that we have a Stackelberg environment and the leader, 1, solved problem (13) which yelded an equilibrium solution such that:

$$U^1_L + U^1_W W_L + U^1_W W_L \, dR^2 / dL_1 = 0 \quad , \tag{29}$$

$$L_2 = R^2(L_1) \quad \text{or} \qquad U^2_L / U^2_W = - W_L \tag{30}$$

and also the labor demand equation.

Let us call this solution L_1^S and $L_2^S = R^2(L_1^S)$.

Assume that $R^2(L_1^S) > M_2$. Then, obviously, $L_2 = M_2$. But in that case the leader considers that the follower, at the margin, does not respond to its quantity. Therefore the leader obeys

$$U^1_L + U^1_W W_L = 0 \quad \text{and} \quad L_2 = M_2 \tag{31}$$

i.e., the leader reacts to M_2 according to its reaction function, being the equilibrium solution:

$$L_1 = R^1(M_2) \quad \text{and} \quad L_2 = M_2 \tag{32}$$

Being $R^2(L_1)$ negatively sloped, if M_2 is close to $R^2(L_1^S)$ we conclude, comparing (29) with (31), that at the new $L_1{}^* = R^1(M_2)$, (29) is negative. Therefore $L_1^S < L_1{}^*$. Also, union 1's utility level will be higher that at L_1^S.

The features of the corner equilibrium are altogether similar to the ones of the Cournot constrained solution - graphically, they coincide. Given, also, that the Stackelberg equilibrium yields higher total employment than the Cournot outcome - once it is on 2's reaction function below it -, the corner solution will imply a smaller total employment than the unconstrained Stackelberg one. If M_2 is small enough, the leader's employment may be higher than in the Stackelberg equilibrium (at the limit, if M_2 is 0, the leader chooses the monopoly union solution; this may imply a higher employment for the leader than the Stackelberg equilibrium); but if not, the leader's employment does not need to be so large as in the unconstrained case.

Proposition 2: For a Stackelberg duopoly, being the leader the large union:

1. 1., 3. and 4. of Proposition 1 hold when there is insufficient

membership of the follower to ensure the interior solution.

2. the large union will behave as a monopolist with respect to the residual demand. It may have smaller employment than in the unconstrained case; and it will attain a higher utility level than in the interior solution.

If $R^2(L_1{}^S) < M_2$, we can have the standard interior Stackelberg solution. But an interesting possibility may occur: it may be worthwhile for the leader to decrease its employment and force the other union to employ all its members. This is the issue taken below.

"Employment-pushing"

Consider that we have the Stackelberg scenario above but that $R^2(L_1{}^S) < M_2$. It may be the case that it is worthwhile for union 1 to establish an $L_1{}^{**} < L_1{}^S$ and make union 2 reach its bound. This allows union 1 to behave as a monopolist with respect to the residual demand.

This strategy is worthwhile iff:

$$U^1\{L_1{}^S, W[L_1{}^S + R^2(L_1{}^S)]\} < U^1\{R^1(M_2), W[R^1(M_2) + M_2]\} \quad (33)$$

Clearly, this can happen iff there is equilibrium stability. Let us see Fig. 3. The unrestricted Stackelberg equilibrium yielded point A, which was possible with union's members, M_2. If $M_2 < M$ - where M is the employment of union 2 that corresponds to the point where 1's indifference curve attained in the Stackelberg solution crosses 1's reaction function -, union 1 may decrease its quantity and still benefit from the fact that 2 can no longer expand its own employment. 1 chooses the point more to the south that touches the new reaction function M_2BN, i.e., the new solution is point C.

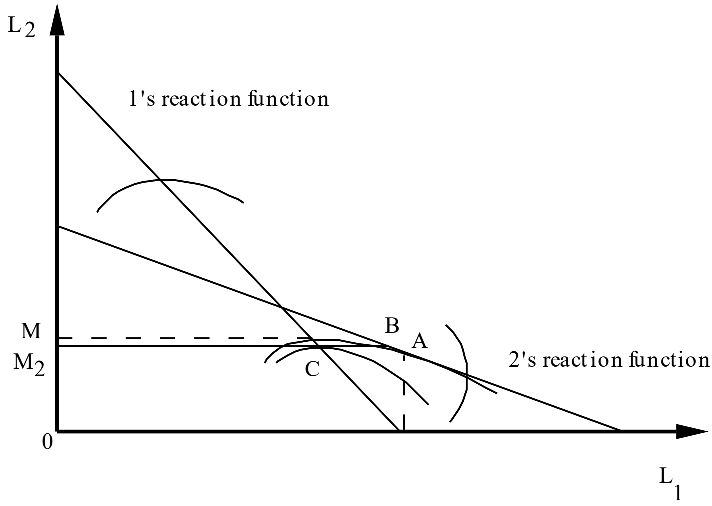

Figure 3.

From the figure we can also conclude that such possibility did not exist if 1 was a Cournot follower...

M solves:

$$U^1\{L_1^S, W[L_1^S + R^2(L_1^S)]\} = U^1\{R^1(M), W[R^1(M) + M]\} \quad (34)$$

Then, if $M_2 > M > R^2(L_1{}^S)$, we have the Stackelberg solution. Being $M > M_2 > R^2(L_1{}^S)$, union 1 decreases its quantity in order to restrict reaction of the opponent. It chooses an $L_1{}^{**}$ such that:

$$L_1{}^{**} = R^1(M_2) \quad (35)$$

In this solution,

$$L = R^1(M_2) + M_2 \quad (36)$$

and therefore, an increase in membership of union 2 will lead to:

$$dL/dM_2 = dR^1/dL_2(M_2) + 1 \quad (37)$$

This will be positive iff:

$$dR^1/dL_2(M_2) > -1 \quad (38)$$

which is guaranteed by (11). Then, an increase of M_2 around this equilibrium will lead to an increase in total employment and a decrease in the equilibrium wage. Total employment will be smaller than in the interior solution: the Cournot outcome originates a smaller total employment than the Stackelberg equilibrium; and point C is on 1's reaction function - with slope smaller than 1 in absolute value - to the right of the Cournot solution and therefore with lower employment than the latter.

Proposition 3: In a Stackelberg duopoly, when membership of the follower, even if sufficient for the interior solution, is near the latter:

1. it may be "profitable" for the union leader to decrease its own employment relative to the Stackelberg outcome in order to be able to behave as a monopolist with respect to residual demand.

2. If the behavior of the leader is the one described in 1. of this proposition, 3. and 4. of Proposition 1. hold.

Notice that whenever one of the unions is pushed to the employment ceiling, a change in the bound will have the same effect on the equilibrium level of total employment. The constrained solution will always imply a smaller total employment and higher wage than the corresponding interior solution.

Also, the large union always benefits from restricting membership of the other union – that is, in professional markets, the recognition of foreign certificates by national institutions may be seen as a means of constraining membership of the "fringe" union. Alternatively, immigration constraints may have the same effect.

Efficient bargaining between unions

Consider the efficient bargaining solution when there are membership bounds. It is clear that if union members could be switched from one union to the other, we would eventually arrive at the interior efficient bargaining solution of section II.3.

Let us instead consider the case in which such behavior is not possible. Assume the small union is 2 - the only union that will be affected by a ceiling (we assume the other is a "large" union). Then the efficient bargaining problem can be stated for the membership ceilings as:

$$\underset{L_1, L_2, W}{\text{Max}} \; [U^1(L_1, W) - \bar{U}^1]^{\circledcirc} \; [U^2(L_2, W) - \bar{U}^2] \qquad (39)$$

$$\text{s.t.:} \quad L_1 + L_2 = L(W) \quad \text{or} \quad W = W(L_1 + L_2) \quad \text{and}$$

$$L_2 \circledcirc M_2$$

The lagrangean of the problem can be stated as:

$$\underset{L_1, L_2}{\text{Max}} \; \{U^1[L_1, W(L_1+L_2)] - \bar{U}^1\}^{\circledcirc}\{U^2[L_2, W(L_1+L_2)] - \bar{U}^2\} + \circledcirc \; (M_2 - L_2) \qquad (40)$$

If the interior solution of the efficient bargaining problem without the restriction originates an $L_2^* < M_2$, we will stay in the interior solution. If not, the corner solution, from first derivative with respect to L_1 - which will hold both in restricted or unrestricted solutions - will yield that, at $L_2 = M_2$, it must be the case that:

$$\circledcirc \; (U^1_L + U^1_W \; W_L) / [U^1 - \bar{U}^1] + W_L \, U^2_W / [U^2 - \bar{U}^2] = 0 \qquad (41)$$

Then this equality determines L_1.

The new solution does not obey either the previously defined efficiency or distribution locus. But we conclude that, if (41) (at least near the relevant range) increases with L_2 (then, at an M_2 smaller than the unrestricted efficient bargaining solution, (41) is negative and the maximand (40) is already decreasing with L_1), L_1 will be smaller than if the restriction was not binding, i.e., than its unrestricted equilibrium level. The opposite will occur if (41) decreases with L_2 - we could not rule this out, once it depends on second derivatives also of labor demand.

It seems more plausible that L_1 should be now smaller than when the bound was not imposed. The intuition for this is that, with efficient bargaining, as union 2 cannot benefit from additional employment, it will try, within the coalition, to compensate by asking a rise in the wage - hence, a decrease in employment of the other union.

On the other hand, we have seen that in most of the Cournot and Stackelberg cases, in the corner solution where one union employs all its members, the other reacts according to its reaction function - behaving as a monopolist with respect to residual demand; then, in the equilibrium:

$$U^1_L + U^1_W W_L = 0 \quad \text{at/and} \quad L_2 = M_2 \tag{42}$$

Looking at (41) - which has the sign of ∂◎/∂L_1, where ◎ denotes the lagrangean (40) -, at M_2 and the L_1 of the solution of (42), the left hand-side - equal in that case to $W_L U^2_W / [U^2 - \bar{U}^2]$ - is negative; this implies that the maximand (40) is already decreasing: the efficient bargaining solution when the membership restriction is active will yield a lower L_1 than the solution of (42).

Proposition 4: With efficient bargaining between the unions and insufficient membership of one of the unions:

1. employment of the union not affected by the ceiling may be lower or higher than in the interior solution.

2. employment of the union not affected by the ceiling is lower than if she reacted as a monopolist with respect to residual demand.

Employed membership requirements

Suppose now that there are "minimum employment" laws: for a union to be legally constituted it must have at least a minimum of $\bar{L}$ employed members. Alternatively, we could interpret such bound as the minimum level of employment the union is willing to accept - as in the general Stone-Geary function. We inquire, below:

- what is the labor market outcome.

- in which conditions will the incumbent(s) engage in entry-deterrence practices.

Cournot duopoly

1. Clearly, if

$$L_2^* = R^2[R^1(L_2^*)] < \bar{L} \tag{43}$$

the best union 2 will be able to do is to set

$$L_2 = \bar{L} \tag{44}$$

Now, we will have that

$$L_1^* = R^1(\bar{L}) \tag{45}$$

Because reaction functions are negatively sloped, 1's employment will decrease, relative to the case with no constraint and both unions will be worse-off in terms of utility.

Let us see the equilibrium solution in Fig. 4., where we consider that union 2 is the one affected by the constraint. 2's reaction function is equal to the old one from the L_2 axis, i.e., from M to B (till $L_2=\bar{L}$) - where the curve shows a kink - and an horizontal line afterwards till point C. At point C, union 2 is indifferent between employing $\bar{L}$ and closing - point C is in the same indifference curve that crosses the old reaction function at the point where $L_2 = 0$; after C, i.e., if $L_1 > D$, union 2 gives up the market and its reaction function continues in DN.

In sum, the effect of the minimum employment restriction on union 2 - assume the other is a "large union" - is to switch its reaction function from MN to MBC - with a kink at B - and with a discontinuity after C, continuing in DN.

The intersection of the two reaction functions switches from A, the interior Cournot equilibrium, to E when the employment restriction is imposed. At E, 1's reaction function crosses the other's new reaction function.

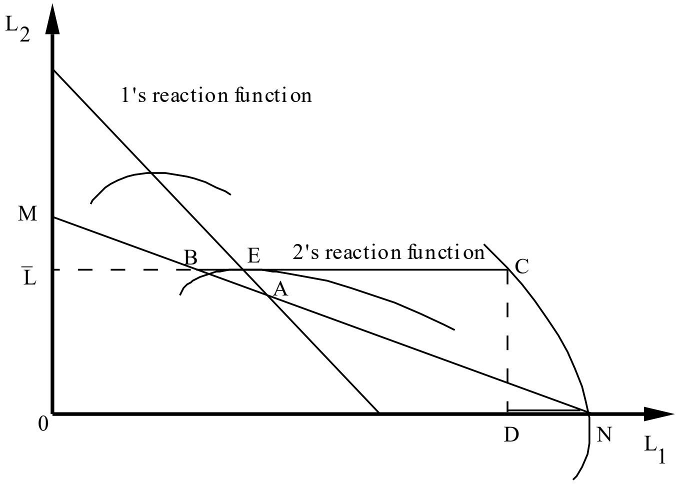

Figure 4.

Notice that in point E, union 2 may be better-off than in point A: as long as $\bar{L}$ is smaller than the level at which union 2's indifference curve that crosses A touches 1's reaction function. Union 1 will always loose utility with the bound.

Total employment will be larger than if the bound was not imposed, once E is to the the North-West of A on 1's reaction function, with slope smaller than 1 in absolute value.

Total employment will be:

$$L = \bar{L} + R^1(\bar{L}) \tag{46}$$

An increase in $\bar{L}$ will increase total employment (decrease the wage rate) iff:

$$1 + dR^1/dL_2(\bar{L}) > 0 \quad , \tag{47}$$

i.e., if,

$$|\, dR^1/dL_2(\bar{L})\, | < 1 \tag{48}$$

It is easy to show that with stability and around L_2^*,

$$R^2(R^1(L_2^*)) = L_2^* < R^2(R^1(\bar{L})) < \bar{L} \tag{49}$$

and therefore, no problem will occur for the solution in the corner.

Proposition 5: If Cournot duopoly implies for a particular union a solution such that its employment is smaller than the membership floor:

1. The union's employment equals the floor (provided it is smaller than its membership). It may attain a higher utility level than in the interior solution.
2. The other union will behave as a monopolist with respect to the residual demand, but will have a lower utility level and smaller employment than in the unconstrained equilibrium.
3. An increase in the membership floor will decrease the other union's employment, increase total employment and decrease the wage.
4. Employment is larger and the wage lower than in the interior solution.

2. If $L_2^* = R^2[R^1(L_2^*)] > \bar{L}$, we expect the Cournot solution to hold.

Stackelberg equilibrium

Consider now that we have a Stackelberg equilibrium. Suppose the leader is union 1, and the constraint is binding for union 2.

1. Let us see the following Fig. 5 below.

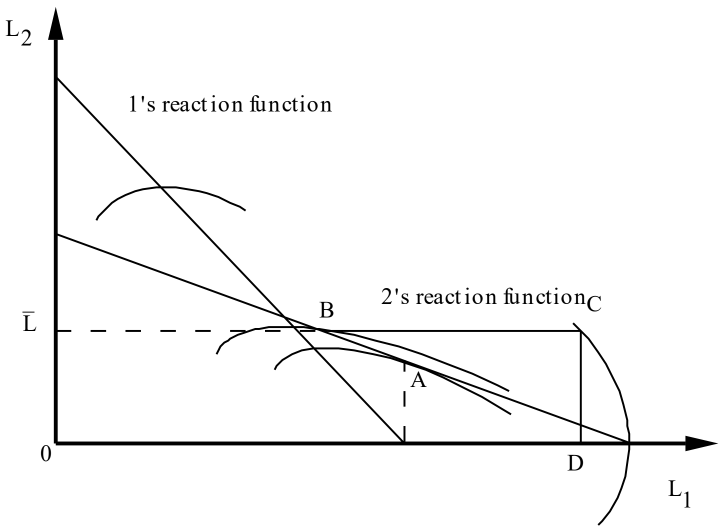

Figure 5.

In the picture, $\bar{L}$ is smaller than the value of L_2 in the Cournot game. Then, the best the Stackelberg leader can now do is to choose the corner B, where 1's employment is L_1^{**} such that:

$$\bar{L} = R^2(L_1^{**}) \tag{50}$$

That is,

$$L_1^{**} = R^2(\bar{L})^{-1} \tag{51}$$

Then total employment is equal to:

$$L = R^2(\bar{L})^{-1} + \bar{L} \quad (52)$$

In this case:

$$dL/d\bar{L} = 1/[dR^2/dL_1(\bar{L})] + 1 \quad (53)$$

This will be negative, if, with 2's reaction function is negatively sloped, 2's reaction function has slope smaller than 1 in absolute value.

From Fig. 5, we also conclude that total employment will be smaller than if the bound was not imposed, once B is to the the North-West of A on 2's reaction function, with slope smaller than 1 in absolute value.

Therefore:

Proposition 6: For a Stackelberg duopoly, if the floor is smaller than the follower solution in a Cournot environment, but larger than the follower's employment under the Stackelberg equilibrium:

1. The leader's solution corresponds to the inverse of the followers reaction function evaluated at the bound $\bar{L}$. The leader will loose utility and lower its employment relative to the unconstrained Stackelberg equilibrium.

2. The follower will employ the amount $\bar{L}$ and attain a higher utility than in the interior solution.

3. An increase in the floor will decrease the leader's employment; it will decrease total employment and increase the wage (till the floor reaches the follower's Cournot solution).

4. Employment is smaller and wage higher than in the interior solution.

2. If $\bar{L}$ is larger than the value of L_2 in the Cournot game, the best the Stackelberg leader can now do is to react according to its reaction function, i.e., to behave as a monopolist with respect to the residual demand, and we have the same equilibrium properties as in the constrained Cournot equilibrium.

Proposition 7: For a Stackelberg duopoly, if the floor is larger than the follower solution in a Cournot environment, the equilibrium will have the same features as the two-follower (constrained) case:

1. The leader reacts according to its reaction function. It will show a lower utility than in the unconstrained maximum.

2. The follower will employ the amount $\bar{L}$ and may attain a higher utility level than in the interior solution.

3. An increase in the floor will decrease the leader's employment; it will increase total employment and decrease the wage.

4. Employment may be larger or smaller and wage lower or higher than in the interior solution.

3. Finally:

Proposition 8: If $\bar{L}$ is lower than the value of L_2 of the Stackelberg game, the membership restriction will be inactive.

Entry-deterrence behavior

Consider now the possibility of union 1 deterring entrance. Let us look at Fig. 5. Union 1 indifference curve that touches point B, crosses the L_1 axis to the left of D. If union 1 chooses L_1 = D union 2 will go out of the market, say, it drives wages to zero. In that case it was not worthwhile. But it can happen something like what is depicted in Fig. 6. In this case, clearly union 1 prefers to deter entrance, once the indiference curve that touches point B is now associated with a lower utility than point D.

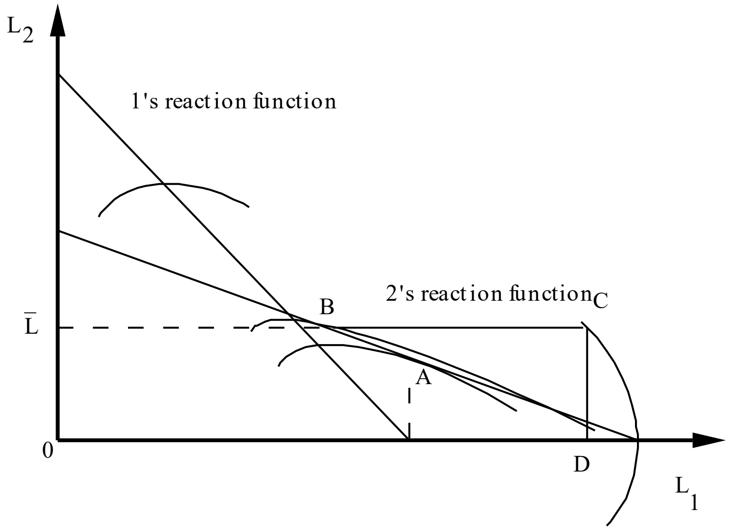

Figure 6.

She will achieve it by setting an L_1 of point D, which is the same as of point C. Point C is, at $L_2 = \bar{L}$, on 2's indifference curve that touches its reaction function at the inactivity level. Then we have that L_1 solves:

$$U^2[\bar{L}, W(L_1 + \bar{L})] = U^2\{0, W[R^2(0)^{-1}]\} \tag{54}$$

If we assume that unions' utility will only be equal to the utility level at 0 employment if (either own employment or) wage is 0, then L_1 must be such that if the potential entrant enters at the required $\bar{L}$, it drives wages to zero[21]. That is, to deter entrance, union 1 has to set :

$$L_1 = L(0) - \bar{L} = L \quad (55)$$

Then, employment decreases with the membership floor. Notice, however, that if both unions are affected by the floor, employment may actually increase when the floor increases. That is, if $L(0) - \bar{L} > \bar{L}$, or $\bar{L} < L(0) / 2$ - otherwise the incumbent will also hit the bound with this policy, and the best it can do is set $L_1 = \bar{L}$...

With (55):

$$W = W[L(0) - \bar{L}] \quad (56)$$

Clearly, this will be profitable iff:

Case A: For $R^2(L_1^S) < \bar{L} < R^2(R^1(L_2^C)) = L_2^C$

This corresponds to the situation of Proposition 6. It is depicted in Fig. 6.

$$U^1\{L(0) - \bar{L}, W[L(0) - \bar{L}]\} > U^1\{R^2(\bar{L})^{-1}, W[R^2(\bar{L})^{-1} + \bar{L}]\} \quad (57)$$

That is, it must yield higher utility than allowing the other union to enter at level $\bar{L}$.

Case B: For $R^2(L_1^S) < R^2(R^1(L_2^C)) = L_2^C < \bar{L}$

This corresponds to the situation of Proposition 7. In this case, the best union 1 could do was to react according to its reaction function. Well, this will yield a lower utility to union 1 than deterring entrance if:

$$U^1\{L(0) - \bar{L}, W[L(0) - \bar{L}]\} > U^1\{R^1(\bar{L}), W[R^1(\bar{L}) + \bar{L}]\} \quad (58)$$

This case is shown in Fig. 7.

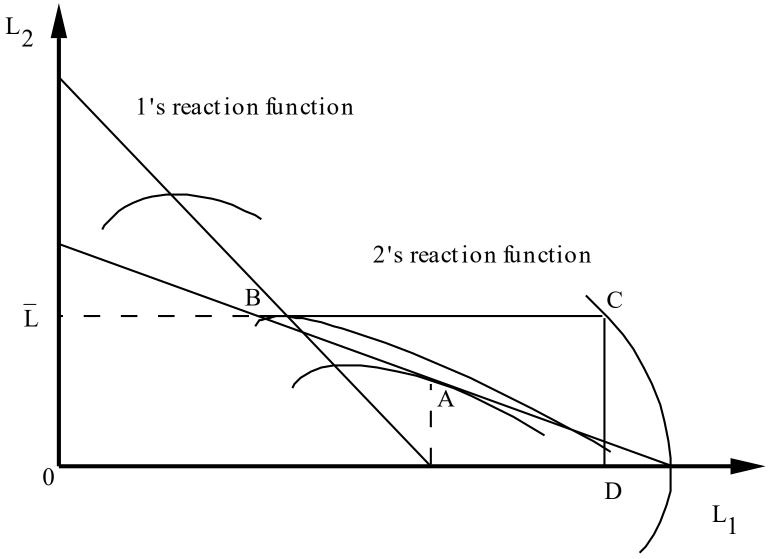

Figure 7.

Case C: For $R^2(L_1^S) > \bar{L}$

This corresponds to the situation of Proposition 8. Then, it may occur that it is more profitable to deter entrance than to allow the other union to enter at the Stackelberg level. It occurs if:

$$U^1\{L(0) - \bar{L}, W[L(0) - \bar{L}]\} > U^1\{L_1^S, W[L_1^S + R^2(L_1^S)]\} \qquad (59)$$

This is depicted in Fig. 8.

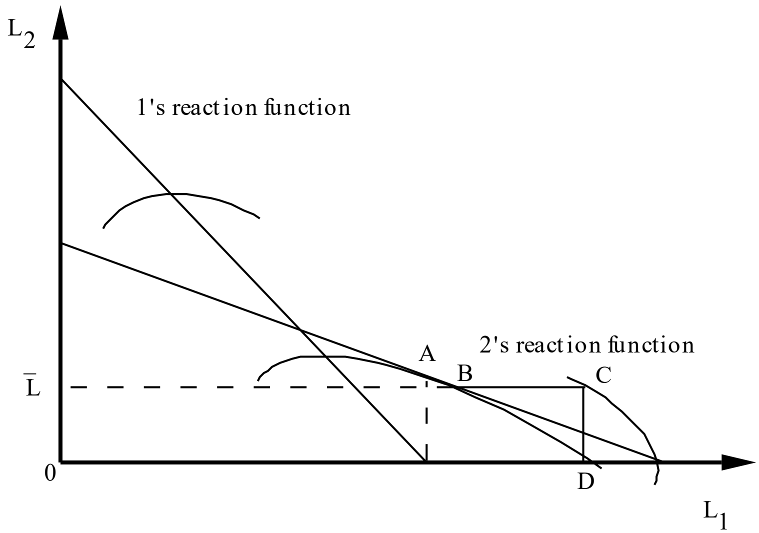

Figure 8.

Entry-deterrence may be more likely when 1 is a leader and 2 a follower. However, it can happen that 1 is a follower - or that it will end up by sharing the market in a Cournot game. Then we have similar conclusions as above, with L_1^S replaced by $L_1^* = R^1(R^2(L_1^*))$ in (59) of case C.

Notice that if we have a Cournot outcome, it may be worthwhile for the incumbent to engage in entry-deterrence even if $\bar{L} = 0$ (this will not occur for a Stackelberg equilibrium).

Proposition 9: 1. Entry deterrence by employment expansion may be profitable for a union leader when there are membership floors.

2. If it is, an increase in the membership floor will decrease total employment (equal to the leader's employment) and increase the wage - unless the leader has reached the floor itself.

3. Propositions 6, 7 and 8 hold with an addition: "provided entry-deterrence is not more profitable".

5.4. Efficient bargaining between unions

If only the second union is affected by the minimum employment requirement, the Nash-maximand becomes:

$$\max_{L_1, L_2, W} \; [U^1(L_1, W) - \bar{U}^1]^{\circledcirc} \; [U^2(L_2, W) - \bar{U}^2] \tag{60}$$

s.t.: $L_1 + L_2 = L(W)$ or $W = W(L_1 + L_2)$

$$L_2 \circledcirc \bar{L}$$

The lagrangean of the problem can be stated as:

$$\max_{L_1, L_2} \; \{U^1[L_1, W(L_1+L_2)] - \bar{U}^1\}^{\delta} \{U^2[L_2, W(L_1+L_2)] - \bar{U}^2\} + \circledcirc(L_2 - \bar{L}) \tag{61}$$

If the interior solution of the efficient bargaining problem without the restriction originates an $L_2^* > \bar{L}$, we will stay in the interior solution. If not, the corner solution, from first derivative with respect to L_1 - which will hold both in restricted or unrestricted solutions - will yield that, at $L_2 = \bar{L}$:

$$\delta\ (U^1_L + U^1_W\ W_L) / [U^1 - \bar{U}^1] + W_L\ U^2_W / [U^2 - \bar{U}^2] = 0 \tag{62}$$

and labor demand applies. (62) determines L_1. Notice that whenever we had that 1 reacted according to its reaction function:

$$U^1_L + U^1_W\ W_L = 0 \tag{63}$$

Therefore at that solution the maximand (61) is already decreasing - the efficient bargaining solution will yield a lower L_1 than if 1 reacted as a monopolist with respect to residual demand.

Proposition 10: With efficient bargaining between the unions, employment of the union not affected by the floor is lower than if the other union behaved as a monopolist towards residual demand - i.e., according to its reaction function with respect to $\bar{L}$.

The chapter extends the framework to model union competition behavior for employment in the presence of employment restrictions that prevent interior solutions. In particular, we analyzed the effects of insufficient membership and of the existence of minimum union employment (or employed membership) rules.

We focussed on the case with two unions and homogeneous labor, and investigate the features of the labor market outcomes when the unions behave as Cournot, Stackelberg or cooperate efficiently.

The main results can be summarized as follows:

1. An employment ceiling affecting one of the unions (or insufficient membership of one of the unions to attain the interior solution) will always benefit the other - no matter if the latter acts as a leader or as a Cournot follower -, which will be able to behave as a monopolist with respect to residual demand and lower its own employment.

If one of the unions is a Stackelberg leader and membership of the other is sufficient for the interior solution, the leader may find it worthwhile to decrease his employment pushing the other's to the bound.

In any case, when one of the union employs all its membership (i.e., is or is forced to the bound), total employment will be lower (the wage higher) than in the interior solution. An increase in that union's members will decrease the other's employment and raise total employment.

With a Stone-Geary utility function ($U^i(L_i,W) = W^{\theta_i} L_i^{(1-\theta_i)}$, $0 < \theta_i < 1$) and linear demand schedule ($W = a - b\,(L_1 + L_2)$) – results summarized in Tables 3 and 4 -, we concluded that insufficient membership is more likely to affect unions with higher preference for employment relative to wage. The cooperative solution will imply a larger employment of the unaffected union, and a smaller total employment (higher equilibrium wage) than in the unconstrained maximum.

2. Minimum employed members′ rules (: a minimum employed membership is required for a union to be considered legal) produce kinks and discontinuities in the unions' reaction functions. In general, these rules may

benefit the union that faces the constraint directly; they will decrease the other union's utility relative to the interior solution. If the equilibrium - Cournot, Stackelberg or cooperative between unions - implies that a follower's interior solution is lower than the floor, his employment is pushed to this floor. An increase of the legal minimum will always increase total employment and decrease the equilibrium wage in a Cournot game; total employment is smaller and wage higher than in the interior solution. However, a change in the floor will decrease total employment (wage) in a Stackelberg game if the floor would allow the Cournot outcome but not the Stackelberg interior solution for the follower; then, total employment is higher and wage lower than in the interior solution.

With homogeneous workers, the existence of high employment floors may induce entry deterrence behavior of the incumbent(s). When entry deterrence is being practiced, an increase in the floor decreases total employment - once it makes the incumbent's behavior less "costly".

With a Stone-Geary utility function and linear demand schedule, we concluded that minimum employment rules are more likely to affect - directly - unions with higher preference for wage relative to employment. Entry deterrence practices seem more likely - "profitable" for the leader - when the follower has low preference for wage relative to employment; in that case, we expect larger total employment and lower wage than if entry deterrence was not engaged. The cooperative solution will imply a smaller employment of the unaffected union, a larger total employment and lower equilibrium wage than in the unconstrained maximum.

Table 1. *Insufficient Membership of Union 2.*

	Conditions	Equilibrium Solution
Cournot	$R^2(R^1(L_2^*))=L_2^* > M_2$	$L_2 = M_2$ $L_1 = R^1(M_2)$
	$R^2(R^1(L_2^*))=L_2^* < M_2$	$R^2(R^1(L_2^*))=L_2^*$ $R^1(R^2(L_1^*))=L_1^*$
	$R^2(L_1^S)=L_2^S > M_2$	$L_2 = M_2$ $L_1 = R^1(M_2)$
Stackelberg	$R^2(L_1^S)=L_2^S < M_2$ and $U^1\{L_1^S, W[L_1^S+R^2(L_1^S)]\} > U^1\{R^1(M_2)$ $, W[R^1(M_2)+M_2]\}$	$R^2(L_1^S)=L_2^S$ $U^1_L / U^1_W = -(1+ dR^2/dL_1)W_L$
Employment - Pushing	$R^2(L_1^S)=L_2^S < M_2$ and $U^1\{L_1^S, W[L_1^S+R^2(L_1^S)]\} < U^1\{R^1(M_2)$ $, W[R^1(M_2)+M_2]\}$	$L_2 = M_2$ $L_1 = R^1(M_2)$
Efficient	$L_2^* > M_2$	$L_2 = M_2$ $\delta\ (U^1_L + U^1_W W_L)/[U^1 - \bar{U}^1] +$ $+ W_L U^2_W/[U^2 - \bar{U}^2] = 0$
Bargaining	$L_2^* < M_2$	$U^1_W/U^1_L + U^2_W/U^2_L = -L_W$ $\delta = \{[U^1(L_1, W) - \bar{U}^1]/$ $/[U^2(L_2, W) - \bar{U}^2]\}\ (U^2_L/U^1_L)$

Table 2. *Insufficient Employment of Union 2.*

	Conditions	Equilibrium Solution
Cournot	$R^2(R^1(L_2^*)) = L_2^* < \bar{L}$	$L_2 = \bar{L}$ $L_1 = R^1(\bar{L})$
	$R^2(R^1(L_2^*)) = L_2^* > \bar{L}$	$R^2(R^1(L_2^*)) = L_2^*$ $R^1(R^2(L_1^*)) = L_1^*$
	$R^2(R^1(L_2^*)) = L_2^* > \bar{L} > L_2^S$	$L_2 = \bar{L}$ L_1 such that $\bar{L} = R^2(L_1)$
Stackelberg	$R^2(R^1(L_2^*)) = L_2^* < \bar{L}$	$L_2 = \bar{L}$ $L_1 = R^1(\bar{L})$
	$\bar{L} < L_2^S$	$R^2(L_1^S) = L_2^S$ $U^1_L / U^1_W = -(1 + dR^2 / dL_1) W_L$
Entry Deterrence	(One for each Stackelberg case)	$L_2 = 0$ $L_1 = L(0) - \bar{L}$
Efficient	$L_2^* < \bar{L}$	$L_2 = \bar{L}$ $\delta (U^1_L + U^1_W W_L) / [U^1 - \bar{U}^1] +$ $+ W_L U^2_W / [U^2 - \bar{U}^2] = 0$
Bargaining	$L_2^* > \bar{L}$	$U^1_W / U^1_L + U^2_W / U^2_L = -L_W$ $\delta = \{[U^1(L_1, W) - \bar{U}^1] /$ $/ [U^2(L_2, W) - \bar{U}^2]\} (U^2_L / U^1_L)$

Table 3. *Insufficient Membership of Union 2 - Stone-Geary Utility and Linear Demand*

	Conditions	Equilibrium Solution
	$(a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)] > M_2$	$L_2 = M_2$ $L_1 = (1-\theta_1)(a/b - M_2)$
Cournot	$(a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)] < M_2$	$L_2^* = (a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)]$ $L_1^* = (a/b)(1-\theta_1)\theta_2 / [1-(1-\theta_1)(1-\theta_2)]$
	$(a/b)(1-\theta_2)\theta_1 > M_2$	$L_2 = M_2$ $L_1 = (1-\theta_1)(a/b - M_2)$
Stackelberg	$M_2 > (a/b)(1-\theta_2\theta^1)$	$L_2^S = \theta_1 a(1-\theta_2)/b$ $L_1^S = a(1-\theta_1)/b$
Employment - Pushing	$(a/b)(1-\theta_2)\theta_1 < M_2$ and $M_2 < (a/b)(1-\theta_2\theta^1)$	$L_2 = M_2$ $L_1 = (1-\theta_1)(a/b - M_2)$
	$a(1-\theta_2)/[b(\theta+1)] > M_2$	$L_2 = M_2$ $L_1 = (1-\theta_1)(a/b - M_2)/(1+\theta_2/\circledcirc)$
Efficient Bargaining	$a(1-\theta_2)/[b(\circledcirc+1)] < M_2$	$L_2 = a(1-\theta_2)/[b(\theta+1)]$ $L_1 = \theta\ a(1-\theta_1)/[b(\theta+1)]$

Table 3.1. *Insufficient Membership of Union 2 - Stone-Geary Utility and Linear Demand*

	Conditions	Equilibrium Solution
Cournot	$(a/b)\,(1-\theta_2)\,\theta_1 /$ $/\,[1-(1-\theta_1)\,(1-\theta_2)] > M_2$	$L = (a/b)\,(1-\theta_1) + \theta_1\, M_2$ $W = \theta_1\,(a - b\,M_2)$
	$(a/b)\,(1-\theta_2)\,\theta_1 /$ $/\,[1-(1-\theta_1)\,(1-\theta_2)] < M_2$	$L = (a/b)\,[(1-\theta_2)\,\theta_1 + (1-\theta_1)\,\theta_2] /$ $/\,[1-(1-\theta_1)\,(1-\theta_2)]$ $W = a\,\theta_1\,\theta_2 / [1-(1-\theta_1)\,(1-\theta_2)]$
Stackelberg	$(a/b)\,(1-\theta_2)\,\theta_1 > M_2$	$L = (a/b)\,(1-\theta_1) + \theta_1\, M_2$ $W = \theta_1\,(a - b\,M_2)$
	$M_2 > (a/b)\,(1-\theta_2\theta^1)$	$L = a(1-\theta_1\,\theta_2)/b$ $W = a\,\theta_1\,\theta_2$
Employment - Pushing	$(a/b)\,(1-\theta_2)\,\theta_1 < M_2$ and $M_2 < (a/b)\,(1-\theta_2\theta^1)$	$L = (a/b)\,(1-\theta_1) + \theta_1\, M_2$ $W = \theta_1\,(a - b\,M_2)$
Efficient Bargaining	$a\,(1-\theta_2) / [\,b\,(\theta+1)] > M_2$	$L = [(a/b)(1-\theta_1) + (\theta_1 + \theta_2/\theta)\,M_2] /$ $/\,(1+\theta_2/\theta)$ $W = (a - b\,M_2)\,(\theta_1 + \theta_2) / (\theta + \theta_2)$
	$a\,(1-\theta_2) / [\,b\,(\theta+1)] < M_2$	$L = a\,[\theta\,(1-\theta_1) + (1-\theta_2)] / [b\,(\theta+1)]$ $W = a\,(\theta_1 + \theta_2) / [b\,(\theta+1)]$

Table 4. *Insufficient Employment of Union 2 - Stone-Geary Utility and Linear Demand*

	Conditions	Equilibrium Solution
	$(a/b)(1-\theta_2)\circledcirc\theta_1 / [1-(1-\theta_1)(1-\theta_2)] < \bar{L}$	$L_2 = \bar{L}$ $L_1 = (1-\theta_1)(a/b - \bar{L})$
Cournot	$(a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)] > \bar{L}$	$L_2^* = (a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)]$ $L_1^* = (a/b)(1-\theta_1)\theta_2 / [1-(1-\theta_1)(1-\theta_2)]$
	$(a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)] > \bar{L} > (a/b)(1-\theta_2)\theta_1$	$L_2 = \bar{L}$ $L_1 = (a/b) - [\bar{L}/(1-\theta_2)]$
Stackelberg	$(a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)] < \bar{L}$	$L_2 = \bar{L}$ $L_1 = (1-\theta_1)(a/b - \bar{L})$
	$\bar{L} < (a/b)(1-\theta_2)\theta_1$	$L_2^S = \theta_1\, a(1-\theta_2)/b$ $L_1^S = a(1-\theta_1)/b$
Entry Deterrence	(One for each Stackelberg case)	$L_2 = 0$ $L_1 = (a/b) - \bar{L}$
Efficient	$a(1-\theta_2)/[b(\theta+1)] < \bar{L}$	$L_2 = \bar{L}$ $L_1 = (1-\theta_1)(a/b - \bar{L})/(1+\theta_2/\theta)$
Bargaining	$a(1-\theta_2)/[b(\theta+1)] > \bar{L}$	$L_2 = a(1-\theta_2)/[b(\theta+1)]$ $L_1 = \theta\, a(1-\theta_1)/[b(\theta+1)]$

Table 4.1. *Insufficient Employment of Union 2 - Stone-Geary Utility and Linear Demand*

	Conditions	Equilibrium Solution
	$(a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)] < \bar{L}$	$L = (a/b)(1-\theta_1) +$ ◎$\theta_1 \bar{L}$ $W = \theta_1 (a - b\bar{L})$
Cournot	$(a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)] > \bar{L}$	$L = (a/b)[(1-\theta_2)\theta_1 + (1-\theta_1)\theta_2] / [1-(1-\theta_1)(1-\theta_2)]$ $W = a\theta_1\theta_2 / [1-(1-\theta_1)(1-\theta_2)]$
	$(a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)] > \bar{L} > (a/b)(1-\theta_2)\theta_1$	$L = (a/b) - [\theta_2/(1-\theta_2)]\bar{L}$ $W = b[\theta_2/(1-\theta_2)]\bar{L}$
Stackelberg	$(a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)] < \bar{L}$	$L = (a/b)(1-\theta_1) + \theta_1\bar{L}$ $W = \theta_1(a - b\bar{L})$
	$\bar{L} < (a/b)(1-\theta_2)\theta_1$	$L = a(1-\theta_1\theta_2)/b$ $W = a\theta_1\theta_2$
Entry Deterrence	(One for each Stackelberg case)	$L = (a/b) - \bar{L}$ $W = b\bar{L}$
Efficient	$a(1-\theta_2)/[b(\theta+1)] < \bar{L}$	$L = [(a/b)(1-\theta_1) + (\theta_1 + \theta_2/\theta)\bar{L}] / (1+\theta_2/\theta)$ $W = (a - b\bar{L})(\theta_1+\theta_2)/(\theta+\theta_2)$
Bargaining	$a(1-\theta_2)/[b(\theta+1)] > \bar{L}$	$L = a[\theta(1-\theta_1) + (1-\theta_2)] / [b(\theta+1)]$ $W = a(\theta_1+\theta_2)/[b(\theta+1)]$

Notes

[1] Citing Rosen (1970) as the first author to recognize strategic interdependency among unions.

[2] Also Davidson (1988), Dixon (1988), Dowrick (1989), Jun (1989) and Dobson (1994), for example, where the effect of the existence of oligopoly in the product market is investigated.

[3] After Cournot (1838).

[4] After Nash (1950).

[5] Von Stackelberg (1934).

[6] In MacDonald & Solow's (1981) lines.

[7] Article 8º, §.2, D.L. 215-B/75, April 30th. Changes in union regulations are subject to similar requirements (10% of associates or 2000 workers) - Article 43º, §.1. "Unions" and Federations require one third of target unions – of the region or cathegory, respectively –, obbeying some majority of affiliated workers criteria, according to Article 8º, §.3. See Bettencourt & Baptista (1999).

[8] See, for example, Spence (1977), Dixit (1979 and 1980), and Schmalensee (1981) for the analysis of entry barriers in the product market, which work similarly to these restrictions.

[9] Even if in a different manner: Dixit's fixed costs introduce discontinuities in the reaction functions while in our case the type of employment constraints we consider produce kinks; discontinuities only arise with minimum employment rules.

[10] See Adnett (1996), p.27.

11 See Prazeres (2001).

12 CGTP-IN and UGT, the two major union confederations – that coordenate activity of ("primary") unions, filliated in "unions" or federations. See Cerdeira (1997), p.57.

13 Founded in 1978. See Cerdeira (1997), p.16, footnote 12.

14 See Cerdeira (1997), p. 83.

15 The reader is referred to Martins & Coimbra (1997) for additional comments on the solutions for two unions. This section summarizes the main results for the general case, needed for the following exposition.

16 Nevertheless, most of the results below would also apply if this function represented the marginal revenue product of labor and if firms did not behave competitively in the product market.

17 Existence is guaranteed by concavity of each union's utility function with respect to L_i, and uniqueness is satisfied if dR^1/dL_2 / $(1+dR^1/dL_2)$ + dR^2/dL_1 / $(1+dR^2/dL_1) < 0$, which will hold if $-1 \leq dR^i / dL_j \leq 0$, i=1,2, j=2,1. This ensures that optimal L_i falls as L rises. See Friedman (1983), p. 30-33.

18 If we departed from a Cournot equilibrium...

19 A reasonable assumption would make θ equal to M_1 / M_2, the number of members of union 1 divided by the number of members of union 2. See Martins & Coimbra (1997 and 1997a) for additional interpretation.

20 Which could be derived from the problem

$$\max_{L_1, L_2, W} \; U^1(L_1,W)$$

$$\text{s.t.:} \quad U^2(L_2,W) \geq \bar{U}^2$$

$$W = W(L_1+L_2)$$

21 Or to the minimum acceptable wage for union 2 to stay in the labor market according to the shape of its utility function, i.e., $W = W[R^2(0)^{-1}]$, being $R^2(0)^{-1}$ the level of L_1 for which union 2 reacts with $L_2 = R^2(L_1) = 0$. For simplicity, we assume it to be 0, i.e., $W[R^2(0)^{-1}] = 0$. A more general formulation would replace L(0) by $[R^2(0)^{-1}]$ in the formulas below.

Appendix

We want to show that in a Cournot duopoly:

If $R^2(R^1(L_2^*)) > M_2$, then, $R^2(R^1(M_2)) > M_2$ (A1)

Given that reaction functions are negatively sloped, we know that

$R^2(R^1(M_2)) < R^2(R^1(L_2))$ if $M_2 < L_2$ (A2)

With a stable equilibrium, it cannot occur, however, that at $L_2^* = R^2(R^1(L_2^*))$:

$R^2(R^1(M_2)) < M_2 < R^2(R^1(L_2^*)) = L_2^*$ (A3)

and it will be the case that:

$M_2 < R^2(R^1(M_2)) < R^2(R^1(L_2^*)) = L_2^*$ (A4)

The reason is that, with stability, a small increase in the argument of $R^2(R^1(.))$ will originate an increase in the function which is smaller than one. For an infinitesimal variation of M_2 around L_2*:

$$R^2(R^1(L_2^*)) - R^2(R^1(M_2)) \approx (dR^2/dL_1)\ (dR^1/dL_2)\ (L_2^* - M_2) \tag{A5}$$

If

$$(dR^2/dL_1)\ (dR^1/dL_2) < 1 \tag{A6}$$

then:

$$(dR^2/dL_1)\ (dR^1/dL_2)\ (L_2^* - M_2) \approx R^2(R^1(L_2^*)) - R^2(R^1(M_2)) < L_2^* - M_2 \tag{A7}$$

Therefore

$$R^2(R^2(L_2^*)) - R^2(R^1(M_2)) + M_2 < L_2^* \tag{A8}$$

Because $R^2(R^1(L_2^*)) = L_2^*$, this implies that:

$$M_2 < R^2(R^1(M_2)) \tag{A9}$$

Therefore, when $M_2 < L_2^* = R^2(R^1(L_2^*))$, we will have the corner equilibrium described by (20).

References

Adnett, N. (1996). *European Labor Markets*. Addison Wesley Longman.

Bertrand, J. (1883). "Review." *Journal des Savants*, 68, 499-508.

Bettencourt, P.O., & Baptista, A.M. (1999). *Legislação do Trabalho.* 5th Edition. Lisboa: Quid Juris.

Binmore, K., Rubinstein, A., & Wolinsky, A. (1986). The Nash bargaining solution in economic modelling. *Rand Journal of Economics,* 17(2), 176-88. doi. 10.2307/2555382

Bowley, L. (1924). *The Mathematical Groundwork of Economics.* Oxford University Press.

Bresnahan, T.F. (1981). Duopoly models with consistent conjectures. *American Economic Review,* 71(5), 507-517.

Calmfors, L. (1993). Centralization of wage bargaining and macroeconomic performance. *Stockholm: Seminar Paper*, No.536. [Retrieved from].

Calmfors, L., & Driffill, J. (1988). Bargaining structure, corporatism and macroeconomic performance. *Economic Policy*, 3(6), 13-61. doi. 10.2307/1344503

Carruth, A.A., & Oswald, A.J. (1987). On union preferences and labour market models: Insiders and outsiders. *Economic Journal*, 97(386), 431-45. doi. 10.2307/2232888

Cerdeira, M.C. (1997). *A Evolução da Sindicalização Portuguesa de 1974 a 1995.* Lisboa: Ministério para a Qualificação e o Emprego, CICT.

Corneo, G.G. (1993). Idiosyncratic wage bargaining in an oligopolistic single market. *Fifth EALE Annual Conference*, 30th September - 3rd October, Maastricht.

Cournot, A. (1838). *Researches into the Mathematical Principles of the Theory of Wealth.* Paris: Hachette.

Davidson, C. (1988). Multiunit bargaining in oligopolistic industries. *Journal of Labor Economics*, 6(3), 397-422. doi. 10.1086/298189

Dixit, A. (1979). A model of duopoly suggesting a theory of entry barriers. *Bell Journal of Economics*, 10(1), 20-32. doi. 10.2307/3003317

Dixit, A. (1980). The role of investment in entry-deterrence. *Economic Journal*, 90(357), 95-106. doi. 10.2307/2231658

Dixon, H. (1988). Unions, oligopoly and the natural range of employment. *Economic Journal*, 98(393), 1127-1147. doi. 10.2307/2233723

Dobson, P.W. (1994). Multifirm unions and the incentive to adopt pattern bargaining in oligopoly. *European Economic Review*, 38(1), 87-100. doi. 10.1016/0014-2921(94)90007-8

Dowrick, S. (1989). Union-oligopoly bargaining. *Economic Journal*, 99(398), 1123-1142. doi. 10.2307/2234092

Dunlop, J.T. (1944). *Wage Determination Under Trade Unions.* New York: Macmillan Co.

Earle, J.S., & Pencavel, J. (1990). Hours of work and trade unionism. *Journal of Labor Economics*, 8(1), S150-174. doi. 10.1086/298248

Farber, H.S. (1986). The analysis of union behavior. In *Handbook of Labor Economics*., Vol II, edited by O. Ashenfelter and R. Layard.

Friedman, J. (1983). *Oligopoly Theory.* Cambridge: Cambridge University.

Friedman, M. (1982). *Capitalism and Freedom.* Chicago: The University of Chicago Press.

Grossman, G.M. (1983). Union wages, temporary lay-offs, and seniority. *American Economic Review*. Reprinted in *Economic Models of Trade Unions*, edited by P. Garonna, P. Mori and P. Tedeschi. Chapman & Hall, 1992.

Gylfason, T., & Lindbeck, A. (1984a). Competing wage claims, cost inflation and capacity utilization. *European Economic Review*, 24(1), 1-21. doi. 10.1016/0014-2921(84)90010-2

Gylfason, T., & Lindbeck, A. (1984b). Union rivalry and wages: An oligopolistic approach. *Economica*, 51(202), 129-139. doi. 10.2307/2554205

Hart, O. (1982). A model of imperfect competition with Keynesian features. *Quarterly Journal of Economics*, 97(1), 109-138. doi. 10.2307/1882629

Hendricks, W.E., & Khan, L.M. (1982). The determinants of bargaining structure in U.S. manufacturing industries. *Industrial and Labor Relations Reviews*, 35(2), 181-195. doi. 10.1177/001979398203500202

Horn, H., & Wolinsky, A. (1988). Worker substitutability and patterns of unionisation. *Economic Journal*, 98(391), 484-497. doi. 10.2307/2233379

Johnson, G.E. (1990). Work rules, featherbedding, and Pareto-optimal union-management bargaining. *Journal of Labor Economics*, 8(1), S237-259. doi. 10.1086/298251

Jun, B.H. (1989). Non-cooperative bargaining and union formation. *Review of Economic Studies*, 56(1), 59-76. doi. 10.2307/2297749

Leontief, W.W. (1946). The pure theory of the guaranteed annual wage contract. *Journal of Political Economy*, 54(1), 76-79. doi. 10.1086/256311

Lewis, H.G. (1959). Competitive and monopoly unionism. *In* P. Bradley, (Ed.), *The Public Stake in Union Power.* (pp.181-208), Charlottesville, Virginia: University of Virginia Press, 181-208.

Lindbeck, A., & Snower, D.J. (1990). *The Insider-Outsider Theory of Employment and Unemployment*. Macmillan.

MaCurdy, T.E., & Pencavel, J.H. (1986). Testing between competing models of wage and employment determination in unionized markets. *Journal of Political Economy*, 94(3), S3-S39. doi. 10.1086/261398

Martin, S. (1993) *Advanced Industrial Economics.* Cambridge: Blackwell.

Martins, A.P., & Coimbra, R. (1997). Union duopoly with homogeneous labor. *Working Paper*, No.84.97. Lisboa, Universidade Católica Portuguesa.

Martins, A.P., & Coimbra, R. (1997a). Union oligopoly and entry in the presence of homogeneous labor. *Working Paper*, No. 85.97. Lisboa, Universidade Católica Portuguesa.

McDonald, I.M., & Solow, R.M. (1981). Wage bargaining and employment. *American Economic Review*, 71(5), 896-908.

Mori, P.A., & Tedeschi, P. (1992). Economic models of trade unions: An introduction. *In Economic Models of Trade Unions*, P. Garonna, P. Mori and P. Tedeschi. Chapman & Hall, 1992.

Nash, J.F. (1950). The bargaining problem. *Econometrica*, 18(2), 155-162. doi. 10.2307/1907266

Nash, J.F. (1953). Two-person cooperative games. *Econometrica*, 21(1), 128-140. doi. 10.2307/1906951

Nickell, S.J., & Andrews, M. (1983). Unions, real wages and employment in Britain 1951-79. *Oxford Economic Papers*, 35, 183-206. doi. 10.1093/oxfordjournals.oep.a041610

Oswald, A.J. (1979). Wage determination in an economy with many trade unions. *Oxford Economic Papers*, 31(3), 369-385. doi. 10.1093/oxfordjournals.oep.a041454

Oswald, A.J. (1982). The microeconomic theory of the trade union. *Economic Journal*, 92, 576-95. doi. 10.1007/BF02299147

Prazeres, F. (2001). Contratação Colectiva. In *Trabalho e Relações Laborais*, edited by António Oliveira das Neves. Oeiras: Celta Editora.

Rosen, S. (1970). Unionism and the occupational wage structure in the United States. *International Economic Review,* 11(2), 269-286. doi. 10.2307/2525669

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica*, 50(1), 97-109. doi. 10.2307/1912531

Schmalensee, R.C. (1981). Economies of scale and barriers to entry. *Journal of Political Economy,* 89(6), 1228-1238. doi. 10.1086/261031

Spence, A.M. (1977). Entry, capacity, investment oligopolistic pricing. *Bell Journal of Economics*, 8(2), 534-544. doi. 10.2307/3003302

Tantarelli, E. (1986). The regulation of inflation and unemployment. *Industrial Relations*, 25(1), 1-15. doi. 10.1111/j.1468-232X.1986.tb00664.x

Ulph, A., & Ulph, D. (1990). Union bargaining: A survey of recent work. In *Current Issues in Labour Economics*, edited by D. Sapsford & Z. Tzannatos, Macmillan.

Varian, H.R. (1978) *Microeconomic Analysis.* New York: Norton.

von Stackelberg, H. (1934). *Marktform und Gleichgewicht*. Vienna: Julius Springer.

Labor Supply, Demand and Institutional Wage Movements

Author: **Ana Paula Martins**
Universidade Católica Portuguesa, Portugal

ISBN: 978-625-7501-20-0 (e-Book)
KSP Books 2021

Ana Paula Martins

Professor Ana Paula Martins studied Economics in Lisbon at Catholic University of Portugal (BA 1982) and concluded a Ph.D. in New York at the Graduate School of Arts and Sciences of Columbia University (1987). She has taught at both schools and at New University of Lisbon graduate and undergraduate courses. Currently, she is Assistant Professor at the Catholic University of Portugal, where she has been teaching Labor Economics and Econometrics. She has published theoretical and applied research articles in Labor Economics and Quantitative Methods in refereed academic journals, and also been presenter and participant in numerous international scientific Conferences on various fields. She was a member of the Catholic University Library Council representing the Economics field.

Professor Martins' other books

Nominal Tales of (for) Real Economies: Taste for Inflation, Trade Money, Collaterals and Reserve Requirements

Social Disruption: Explorations on the Pattern of Criminal and Violent Occurrences in Response to Macroeconomic Conditions

Wages, Wealth and Growth: with Portugal case

Teaching and Evaluation Patterns of a Business BA Program – Teaching Mix and Student Performance

Labor Supply, Demand and Institutional Wage Movements

KSP Books

e-ISBN
978-625-7501-20-0